

Numbers and Notations

© Carmi Merimovich

The Academic College of Tel-Aviv

January 29, 2025

- We all believe we know what a number is
- Programs are full of numbers
- At the end, a program is a list of numbers
- We are being told computers use the binary method
- There are even hints of this ($\ll$, $\gg$, $\&$, $|$, etc...)
- So we need to see how (if!) this is done

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

The Natural Numbers ($\mathbb{N}$)

*Die ganzen Zahlen hat der liebe Gott gemacht,
alles andere ist Menschenwerk*

*[Natural numbers were created by God, every-
thing else is the work of men]
(attributed to) Kronecker*

Whoever is happy with the naturals can skip to [notation](#)

$\mathbb{N}$

- Peano
- Caveperson
- Addition
- Subtraction
- Multiplication
- Switching radices
- Hardware

$\mathbb{Z}$

- Complement
- Hardware

$\mathbb{Q}$

- Examples
- Hardware
- Fixed point
- BCD
- Floating-point
- Examples

- Nothing is **natural** with the natural numbers
- Intuition has been distilled in us from youth
- Intuition is useful, but it might lead to mistakes
- Formalization of the intuition saves problems

▶ extra

$\mathbb{N}$

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

$\mathbb{Z}$

Complement
Hardware

$\mathbb{Q}$

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

- 0 is a constant symbol and S is a unary operator
- 1. $S(n) = S(m) \implies n = m$
 2. For each n , $S(n) \neq 0$
 3. For a set P , if
 - ▶ $0 \in P$ and
 - ▶ for each n , $n \in P \implies S(n) \in P$then P is the set of all the naturals

Intuition

- 0 is the symbol for the minimal natural
- $S(n)$ is $n + 1$ (we have neither $+$ nor 1 as of yet)

$\mathbb{N}$

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

$\mathbb{Z}$

Complement

Hardware

$\mathbb{Q}$

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Definition ($+$)

$$m + 0 = m$$

$$m + S(n) = S(m + n)$$

Definition ($\times$)

$$m \times 0 = 0$$

$$m \times S(n) = m \times n + m$$

Definition (x^y)

$$m^0 = S(0)$$

$$m^{S(n)} = m^n \times m$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Definition

- $n \geq m \iff \exists k \ n = m + k$
- $n \leq m \iff m \geq n$
- $n > m \iff n \geq m \text{ and } n \neq m$
- $n < m \iff m > n$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Logicians, please have mercy **extra**

We are well aware of the following:

- The non-categoricity when using FO induction
- The categoricity when using SO induction
- The problem with the operators definition in FO logic

Rest of humanity

All is perfectly well

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Clear, simple, and **useless** in every day use:

$$0, S(0), S(S(0)), S(S(S(0))), \dots$$

Using Peano axioms we can prove things like

- $S(S(0)) + S(S(S(0))) = S(S(S(S(S(0)))))$
- $S(S(0)) \times S(S(S(0))) = S(S(S(S(S(S(0))))))$

Everything taught at school can be **proved**

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

- Several notations for numbers were devised over time
- Humanity settled on the (decimal) positional notation
- The **decimal** is the less important fact
- The **position** idea is very important
- Mere humans can (could?) **compute** using:
 - ▶ Addition table
 - ▶ Multiplication table

Addition in different methods

Roman	Decimal
$V + V = X$	$5 + 5 = 10$
$XV + XV = XXX$	$15 + 15 = 30$
$XXV + XV = XL$	$25 + 15 = 40$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

(Failing to explain) **Our** Decimal Notation **extra**

We take for granted the alphabet 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

Positioning method

Example: 235

- We tend to say that 235 is a natural number
- However, 235 is definitely not a number
- It is a concatenation of the symbols '2', '3' and '5'
- We explain: We **mean** $200 + 30 + 5$
- Same problem. 200, 30 and 5 are strings
- We mean $2 \times 10^2 + 3 \times 10 + 5$
- Same problem. 10 is a string.

Luckily we have the useless notation!

And a useful theorem

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Theorem

Let x and b be natural numbers such that $b > S(0)$. Then there are unique natural numbers $q < x$ and $d < b$ such that

$$x = q \times b + d.$$

Corollary

Let x and b be natural numbers such that $b > S(0)$. Then there is a unique tuple $\langle d_{n-1}, \dots, d_0 \rangle$, where $d_i < b$ for each $i < n$, such that $x = d_{n-1} \times b^{n-1} + \dots + d_1 \times b^1 + d_0 \times b^0$.

Definition

$\langle d_{n-1}, \dots, d_0 \rangle$ is the b -radix notation for x and we write

$$x = (d_{n-1} \cdots d_0)_b.$$

The default radix is $S(S(S(S(S(S(S(S(S(0))))))))))$.

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Definition

$$\begin{array}{l|l} 1 = S(0) & A = S(9) \\ 2 = S(1) & B = S(A) \\ 3 = S(2) & C = S(B) \\ 4 = S(3) & D = S(C) \\ 5 = S(4) & E = S(D) \\ 6 = S(5) & F = S(E) \\ 7 = S(6) & \\ 8 = S(7) & \\ 9 = S(8) & \end{array}$$

That is it

The number notation we are used to has just been resurrected

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

- Later on we will talk about ‘decimal numbers’
- There are **no** decimal numbers
- There is a decimal **notation** or decimal **representation**
- By which we mean a notation using the alphabet
$$\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

as defined previously

- We have intuition regarding this alphabet

Ditto for ‘binary numbers’, ‘hex numbers’, or whatever

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Examples



$$S(5) = 0 \times S(9) + 6 \times S(0),$$

$$S(9) = 1 \times S(9) + 0 \times S(0).$$

- Thus in radix- $S(9)$ string notation we have

$$S(5) = 6,$$

$$S(9) = 10.$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Arithmetic Operations in $\mathbb{N}$

(Using the positional notation)

$\mathbb{N}$

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

$\mathbb{Z}$

Complement

Hardware

$\mathbb{Q}$

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

- The operations $+$ and $\times$ have been defined
- Positional notation has been defined
- We define $+_b$ and $\times_b$ between **notations**
- Then we identify $+$ with $+_b$ and $\times$ with $\times_b$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

$+$ and $+_b$, the crux of the matter **extra**

- $+_b$ is how long addition is being taught at school
- $+$ is how mathematician defines addition
- Both methods leads to the same result (a theorem!)
- The $+$ definition allows us to prove properties
- With $+_b$ we really take for granted everything

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Definition (Long addition)

Given two notations $\langle d_{n-1}^0 \cdots d_0^0 \rangle$ and $\langle d_{n-1}^1 \cdots d_0^1 \rangle$, their b -radix long addition is defined to be

$$\langle c_n e_{n-1} \cdots e_0 \rangle = \langle d_{n-1}^0 \cdots d_0^0 \rangle +_b \langle d_{n-1}^1 \cdots d_0^1 \rangle,$$

where

$$c_0 = 0,$$

$$d_i = d_i^0 + d_i^1 + c_i,$$

$$c_{i+1} = \begin{cases} 0 & d_i < b, \\ 1 & d_i \geq b \end{cases}$$

and

$$e_i = d_i - c_{i+1} \times b,$$

for each $i < n$.

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

The Long Addition Lemma **extra**

Lemma

Assume $m = (d_{n-1}^0 \cdots d_0^0)_b$ and $n = (d_{n-1}^1 \cdots d_0^1)_b$. Then

$$m + n = (\langle d_{n-1}^0 \cdots d_0^0 \rangle +_b \langle d_{n-1}^1 \cdots d_0^1 \rangle)_b.$$

Worth noticing **extra**

- We can assume the notations have the same width
- Adding two n -digits numbers might yield $n + 1$ -digits

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Proof of the Long Addition Lemma (1) **extra**

Begin with the obvious:

$$\sum_{i=0}^{n-1} d_i^0 \times b^i + \sum_{i=0}^{n-1} d_i^1 \times b^i = \sum_{i=0}^{n-1} (d_i^0 + d_i^1) \times b^i$$

This does not yield a b -radix notation since we might have i 's for which $d_i^0 + d_i^1 \geq b$.

We use the e_i and c_i notation from the definition of $+_b$ and prove by induction that for each $k \leq n$,

$$m + n = \sum_{i=0}^{k-1} e_i \times b^i + c_k \times b^k + \sum_{i=k}^{n-1} (d_i^0 + d_i^1) \times b^i.$$

The point is that $e_i < b$ and $c_i < 2$, thus when $k = n$ the theorem is proved.

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Proof of the Long Addition Lemma (2) extra

The case $k = 0$ is degenerate. Assuming $k < n$ we prove the case $k + 1$.

$$\begin{aligned} \sum_{i=0}^{n-1} (d_i^0 + d_i^1) \times b^i &= \\ &= \sum_{i=0}^{k-1} e_i \times b^i + c_k \times b^k + \sum_{i=k}^{n-1} (d_i^0 + d_i^1) \times b^i = \\ &= \sum_{i=0}^{k-1} e_i \times b^i + c_k \times b^k + (d_k^0 + d_k^1) \times b^k - \\ &\quad c_{k+1} \times b^{k+1} + c_{k+1} \times b^{k+1} + \\ &\quad \sum_{i=k+1}^{n-1} (d_i^0 + d_i^1) \times b^i = \end{aligned}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Proof of the Long Addition Lemma (3) extra

$$\begin{aligned} &= \sum_{i=0}^{k-1} e_i \times b^i + (d_k^0 + d_k^1 + c_k - c_{k+1} \times b) \times b^k \\ &\quad + c_{k+1} \times b^{k+1} + \sum_{i=k+1}^{n-1} (d_i^0 + d_i^1) \times b^i = \\ &= \sum_{i=0}^{k-1} e_i \times b^i + e_k \times b^k + c_{k+1} \times b^{k+1} + \\ &\quad \sum_{i=k+1}^{n-1} (d_i^0 + d_i^1) \times b^i = \\ &= \sum_{i=0}^k e_i \times b^i + c_{k+1} \times b^{k+1} + \sum_{i=k+1}^{n-1} (d_i^0 + d_i^1) \times b^i. \quad \square \end{aligned}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

The Long Addition Lemma, **Meaning**

Numbers and
Notations

©C.M.

Addition table for digits is enough

Addition of large numbers, no matter how large, can be calculated using a small addition table

This was a serious breakthrough!

N

- Peano
- Caveperson
- Addition
- Subtraction
- Multiplication
- Switching radices
- Hardware

Z

- Complement
- Hardware

Q

- Examples
- Hardware
- Fixed point
- BCD
- Floating-point
- Examples

Addition Tables (Binary and Radix-3)

Binary

+	0	1
0	0	1
1	1	10

Radix-3

+	0	1	2
0	0	1	2
1	1	2	10
2	2	10	11

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Addition Table (Radix-12)

+	0	1	2	3	4	5	6	7	8	9	A	B
0	0	1	2	3	4	5	6	7	8	9	A	B
1	1	2	3	4	5	6	7	8	9	A	B	10
2	2	3	4	5	6	7	8	9	A	B	10	11
3	3	4	5	6	7	8	9	A	B	10	11	12
4	4	5	6	7	8	9	A	B	10	11	12	13
5	5	6	7	8	9	A	B	10	11	12	13	14
6	6	7	8	9	A	B	10	11	12	13	14	15
7	7	8	9	A	B	10	11	12	13	14	15	16
8	8	9	A	B	10	11	12	13	14	15	16	17
9	9	A	B	10	11	12	13	14	15	16	17	18
A	A	B	10	11	12	13	14	15	16	17	18	19
B	B	10	11	12	13	14	15	16	17	18	19	1A

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Addition Examples

Decimal (Well known!)

$$\begin{array}{r} 5678 \\ + 1234 \\ \hline \end{array} \quad \begin{array}{r} 5678 \\ + 6331 \\ \hline \end{array}$$

Radix-12

$$\begin{array}{r} (89AB)_{12} \\ + (23A4)_{12} \\ \hline \end{array} \quad \begin{array}{r} (89AB)_{12} \\ + (ABBA)_{12} \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Addition Examples

Decimal (Well known!)

$$\begin{array}{r} 0 \\ 5678 \\ + 1234 \\ \hline \end{array} \quad \begin{array}{r} 5678 \\ + 6331 \\ \hline \end{array}$$

Radix-12

$$\begin{array}{r} (89AB)_{12} \\ + (23A4)_{12} \\ \hline \end{array} \quad \begin{array}{r} (89AB)_{12} \\ + (ABBA)_{12} \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Addition Examples

Decimal (Well known!)

$$\begin{array}{r} \\ 5678 \\ + 1234 \\ \hline 2 \end{array} \quad \begin{array}{r} 5678 \\ + 6331 \\ \hline \end{array}$$

Radix-12

$$\begin{array}{r} (89AB)_{12} \\ + (23A4)_{12} \\ \hline \end{array} \quad \begin{array}{r} (89AB)_{12} \\ + (ABBA)_{12} \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Addition Examples

Decimal (Well known!)

$$\begin{array}{r} \\ 5678 \\ + 1234 \\ \hline 12 \end{array} \quad \begin{array}{r} 5678 \\ + 6331 \\ \hline \end{array}$$

Radix-12

$$\begin{array}{r} (89AB)_{12} \\ + (23A4)_{12} \\ \hline \end{array} \quad \begin{array}{r} (89AB)_{12} \\ + (ABBA)_{12} \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Addition Examples

Decimal (Well known!)

$$\begin{array}{r} 110 \\ 5678 \\ + 1234 \\ \hline 912 \end{array} \qquad \begin{array}{r} 5678 \\ + 6331 \\ \hline \end{array}$$

Radix-12

$$\begin{array}{r} (89AB)_{12} \\ + (23A4)_{12} \\ \hline \end{array} \qquad \begin{array}{r} (89AB)_{12} \\ + (ABBA)_{12} \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Addition Examples

Decimal (Well known!)

$$\begin{array}{r} 00110 \\ 5678 \\ + 1234 \\ \hline 6912 \end{array} \quad \begin{array}{r} 5678 \\ + 6331 \\ \hline \end{array}$$

Radix-12

$$\begin{array}{r} (89AB)_{12} \\ + (23A4)_{12} \\ \hline \end{array} \quad \begin{array}{r} (89AB)_{12} \\ + (ABBA)_{12} \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Addition Examples

Decimal (Well known!)

$$\begin{array}{r} 00110 \\ 5678 \\ + 1234 \\ \hline 6912 \end{array} \qquad \begin{array}{r} 0 \\ 5678 \\ + 6331 \\ \hline \end{array}$$

Radix-12

$$\begin{array}{r} (89AB)_{12} \\ + (23A4)_{12} \\ \hline \end{array} \qquad \begin{array}{r} (89AB)_{12} \\ + (ABBA)_{12} \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Addition Examples

Decimal (Well known!)

$$\begin{array}{r} 00110 \\ 5678 \\ + 1234 \\ \hline 6912 \end{array} \quad \begin{array}{r} 00 \\ 5678 \\ + 6331 \\ \hline 9 \end{array}$$

Radix-12

$$\begin{array}{r} (89AB)_{12} \\ + (23A4)_{12} \\ \hline \end{array} \quad \begin{array}{r} (89AB)_{12} \\ + (ABBA)_{12} \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Addition Examples

Decimal (Well known!)

$$\begin{array}{r} 00110 \\ 5678 \\ + 1234 \\ \hline 6912 \end{array} \quad \begin{array}{r} 100 \\ 5678 \\ + 6331 \\ \hline 09 \end{array}$$

Radix-12

$$\begin{array}{r} (89AB)_{12} \\ + (23A4)_{12} \\ \hline \end{array} \quad \begin{array}{r} (89AB)_{12} \\ + (ABBA)_{12} \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Addition Examples

Decimal (Well known!)

$$\begin{array}{r} 00110 \\ 5678 \\ + 1234 \\ \hline 6912 \end{array} \qquad \begin{array}{r} 1100 \\ 5678 \\ + 6331 \\ \hline 009 \end{array}$$

Radix-12

$$\begin{array}{r} (89AB)_{12} \\ + (23A4)_{12} \\ \hline \end{array} \qquad \begin{array}{r} (89AB)_{12} \\ + (ABBA)_{12} \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Addition Examples

Decimal (Well known!)

$$\begin{array}{r} 00110 \\ 5678 \\ + 1234 \\ \hline 6912 \end{array} \qquad \begin{array}{r} 11100 \\ 5678 \\ + 6331 \\ \hline 2009 \end{array}$$

Radix-12

$$\begin{array}{r} (89AB)_{12} \\ + (23A4)_{12} \\ \hline \end{array} \qquad \begin{array}{r} (89AB)_{12} \\ + (ABBA)_{12} \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Addition Examples

Decimal (Well known!)

$$\begin{array}{r} 00110 \\ 5678 \\ + 1234 \\ \hline 6912 \end{array} \qquad \begin{array}{r} 11100 \\ 5678 \\ + 6331 \\ \hline 12009 \end{array}$$

Radix-12

$$\begin{array}{r} (89AB)_{12} \\ + (23A4)_{12} \\ \hline \end{array} \qquad \begin{array}{r} (89AB)_{12} \\ + (ABBA)_{12} \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Addition Examples

Decimal (Well known!)

$$\begin{array}{r} 00110 \\ 5678 \\ + 1234 \\ \hline 6912 \end{array} \quad \begin{array}{r} 11100 \\ 5678 \\ + 6331 \\ \hline 12009 \end{array}$$

Radix-12

$$\begin{array}{r} 0 \\ (89AB)_{12} \\ + (23A4)_{12} \\ \hline ()_{12} \end{array} \quad \begin{array}{r} (89AB)_{12} \\ + (ABBA)_{12} \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Addition Examples

Decimal (Well known!)

$$\begin{array}{r} 00110 \\ 5678 \\ + 1234 \\ \hline 6912 \end{array} \quad \begin{array}{r} 11100 \\ 5678 \\ + 6331 \\ \hline 12009 \end{array}$$

Radix-12

$$\begin{array}{r} 10 \\ (89AB)_{12} \\ + (23A4)_{12} \\ \hline (3)_{12} \end{array} \quad \begin{array}{r} (89AB)_{12} \\ + (ABBA)_{12} \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Addition Examples

Decimal (Well known!)

$$\begin{array}{r} 00110 \\ 5678 \\ + 1234 \\ \hline 6912 \end{array} \quad \begin{array}{r} 11100 \\ 5678 \\ + 6331 \\ \hline 12009 \end{array}$$

Radix-12

$$\begin{array}{r} 110 \\ (89AB)_{12} \\ + (23A4)_{12} \\ \hline (93)_{12} \end{array} \quad \begin{array}{r} (89AB)_{12} \\ + (ABBA)_{12} \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Addition Examples

Decimal (Well known!)

$$\begin{array}{r} 00110 \\ 5678 \\ + 1234 \\ \hline 6912 \end{array} \quad \begin{array}{r} 11100 \\ 5678 \\ + 6331 \\ \hline 12009 \end{array}$$

Radix-12

$$\begin{array}{r} 1110 \\ (89AB)_{12} \\ + (23A4)_{12} \\ \hline (193)_{12} \end{array} \quad \begin{array}{r} (89AB)_{12} \\ + (ABBA)_{12} \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Addition Examples

Decimal (Well known!)

$$\begin{array}{r} 00110 \\ 5678 \\ + 1234 \\ \hline 6912 \end{array} \quad \begin{array}{r} 11100 \\ 5678 \\ + 6331 \\ \hline 12009 \end{array}$$

Radix-12

$$\begin{array}{r} 01110 \\ (89AB)_{12} \\ + (23A4)_{12} \\ \hline (B193)_{12} \end{array} \quad \begin{array}{r} (89AB)_{12} \\ + (ABBA)_{12} \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Addition Examples

Decimal (Well known!)

$$\begin{array}{r} 00110 \\ 5678 \\ + 1234 \\ \hline 6912 \end{array} \qquad \begin{array}{r} 11100 \\ 5678 \\ + 6331 \\ \hline 12009 \end{array}$$

Radix-12

$$\begin{array}{r} 01110 \\ (89AB)_{12} \\ + (23A4)_{12} \\ \hline (B193)_{12} \end{array} \qquad \begin{array}{r} 0 \\ (89AB)_{12} \\ + (ABBA)_{12} \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Addition Examples

Decimal (Well known!)

$$\begin{array}{r} 00110 \\ 5678 \\ + 1234 \\ \hline 6912 \end{array} \quad \begin{array}{r} 11100 \\ 5678 \\ + 6331 \\ \hline 12009 \end{array}$$

Radix-12

$$\begin{array}{r} 01110 \\ (89AB)_{12} \\ + (23A4)_{12} \\ \hline (B193)_{12} \end{array} \quad \begin{array}{r} 10 \\ (89AB)_{12} \\ + (ABBA)_{12} \\ \hline (9)_{12} \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Addition Examples

Decimal (Well known!)

$$\begin{array}{r} 00110 \\ 5678 \\ + 1234 \\ \hline 6912 \end{array} \quad \begin{array}{r} 11100 \\ 5678 \\ + 6331 \\ \hline 12009 \end{array}$$

Radix-12

$$\begin{array}{r} 01110 \\ (89AB)_{12} \\ + (23A4)_{12} \\ \hline (B193)_{12} \end{array} \quad \begin{array}{r} 110 \\ (89AB)_{12} \\ + (ABBA)_{12} \\ \hline (A9)_{12} \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Addition Examples

Decimal (Well known!)

$$\begin{array}{r} 00110 \\ 5678 \\ + 1234 \\ \hline 6912 \end{array} \quad \begin{array}{r} 11100 \\ 5678 \\ + 6331 \\ \hline 12009 \end{array}$$

Radix-12

$$\begin{array}{r} 01110 \\ (89AB)_{12} \\ + (23A4)_{12} \\ \hline (B193)_{12} \end{array} \quad \begin{array}{r} 1110 \\ (89AB)_{12} \\ + (ABBA)_{12} \\ \hline (9A9)_{12} \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Addition Examples

Decimal (Well known!)

$$\begin{array}{r} 00110 \\ 5678 \\ + 1234 \\ \hline 6912 \end{array} \quad \begin{array}{r} 11100 \\ 5678 \\ + 6331 \\ \hline 12009 \end{array}$$

Radix-12

$$\begin{array}{r} 01110 \\ (89AB)_{12} \\ + (23A4)_{12} \\ \hline (B193)_{12} \end{array} \quad \begin{array}{r} 11110 \\ (89AB)_{12} \\ + (ABBA)_{12} \\ \hline (79A9)_{12} \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Addition Examples

Decimal (Well known!)

$$\begin{array}{r} 00110 \\ 5678 \\ + 1234 \\ \hline 6912 \end{array} \quad \begin{array}{r} 11100 \\ 5678 \\ + 6331 \\ \hline 12009 \end{array}$$

Radix-12

$$\begin{array}{r} 01110 \\ (89AB)_{12} \\ + (23A4)_{12} \\ \hline (B193)_{12} \end{array} \quad \begin{array}{r} 11110 \\ (89AB)_{12} \\ + (ABBA)_{12} \\ \hline (179A9)_{12} \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

More Addition Examples

Radix-3

$$\begin{array}{r} (1202)_3 \\ + (222)_3 \\ \hline \end{array} \quad \begin{array}{r} (1202)_3 \\ + (1120)_3 \\ \hline \end{array}$$

Binary

$$\begin{array}{r} (1011)_2 \\ + (11)_2 \\ \hline \end{array} \quad \begin{array}{r} (1011)_2 \\ + (111)_2 \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

More Addition Examples

Radix-3

$$\begin{array}{r} \\ (1202)_3 \\ + (222)_3 \\ \hline ()_3 \end{array} \quad \begin{array}{r} (1202)_3 \\ + (1120)_3 \\ \hline \end{array}$$

Binary

$$\begin{array}{r} (1011)_2 \\ + (11)_2 \\ \hline \end{array} \quad \begin{array}{r} (1011)_2 \\ + (111)_2 \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

More Addition Examples

Radix-3

$$\begin{array}{r} 0 \\ (1202)_3 \\ + (222)_3 \\ \hline (1)_3 \end{array} \quad \begin{array}{r} (1202)_3 \\ + (1120)_3 \\ \hline \end{array}$$

Binary

$$\begin{array}{r} (1011)_2 \\ + (11)_2 \\ \hline \end{array} \quad \begin{array}{r} (1011)_2 \\ + (111)_2 \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

More Addition Examples

Radix-3

$$\begin{array}{r} 10 \\ (1202)_3 \\ + (222)_3 \\ \hline (01)_3 \end{array} \qquad \begin{array}{r} (1202)_3 \\ + (1120)_3 \\ \hline \end{array}$$

Binary

$$\begin{array}{r} (1011)_2 \\ + (11)_2 \\ \hline \end{array} \qquad \begin{array}{r} (1011)_2 \\ + (111)_2 \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

More Addition Examples

Radix-3

$$\begin{array}{r} 1110 \\ (1202)_3 \\ + (222)_3 \\ \hline (201)_3 \end{array} \quad \begin{array}{r} (1202)_3 \\ + (1120)_3 \\ \hline \end{array}$$

Binary

$$\begin{array}{r} (1011)_2 \\ + (11)_2 \\ \hline \end{array} \quad \begin{array}{r} (1011)_2 \\ + (111)_2 \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

More Addition Examples

Radix-3

$$\begin{array}{r} 01110 \\ (1202)_3 \\ + (222)_3 \\ \hline (2201)_3 \end{array} \quad \begin{array}{r} (1202)_3 \\ + (1120)_3 \\ \hline \end{array}$$

Binary

$$\begin{array}{r} (1011)_2 \\ + (11)_2 \\ \hline \end{array} \quad \begin{array}{r} (1011)_2 \\ + (111)_2 \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

More Addition Examples

Radix-3

$$\begin{array}{r} 01110 \\ (1202)_3 \\ + (222)_3 \\ \hline (2201)_3 \end{array} \qquad \begin{array}{r} 0 \\ (1202)_3 \\ + (1120)_3 \\ \hline \end{array}$$

Binary

$$\begin{array}{r} (1011)_2 \\ + (11)_2 \\ \hline \end{array} \qquad \begin{array}{r} (1011)_2 \\ + (111)_2 \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

More Addition Examples

Radix-3

$$\begin{array}{r} 01110 \\ (1202)_3 \\ + (222)_3 \\ \hline (2201)_3 \end{array} \qquad \begin{array}{r} 00 \\ (1202)_3 \\ + (1120)_3 \\ \hline (2)_3 \end{array}$$

Binary

$$\begin{array}{r} (1011)_2 \\ + (11)_2 \\ \hline \end{array} \qquad \begin{array}{r} (1011)_2 \\ + (111)_2 \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

More Addition Examples

Radix-3

$$\begin{array}{r} 01110 \\ (1202)_3 \\ + (222)_3 \\ \hline (2201)_3 \end{array} \qquad \begin{array}{r} 000 \\ (1202)_3 \\ + (1120)_3 \\ \hline (22)_3 \end{array}$$

Binary

$$\begin{array}{r} (1011)_2 \\ + (11)_2 \\ \hline \end{array} \qquad \begin{array}{r} (1011)_2 \\ + (111)_2 \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

More Addition Examples

Radix-3

$$\begin{array}{r} 01110 \\ (1202)_3 \\ + (222)_3 \\ \hline (2201)_3 \end{array} \qquad \begin{array}{r} 1000 \\ (1202)_3 \\ + (1120)_3 \\ \hline (022)_3 \end{array}$$

Binary

$$\begin{array}{r} (1011)_2 \\ + (11)_2 \\ \hline \end{array} \qquad \begin{array}{r} (1011)_2 \\ + (111)_2 \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

More Addition Examples

Radix-3

$$\begin{array}{r} 01110 \\ (1202)_3 \\ + (222)_3 \\ \hline (2201)_3 \end{array} \qquad \begin{array}{r} 11000 \\ (1202)_3 \\ + (1120)_3 \\ \hline (0022)_3 \end{array}$$

Binary

$$\begin{array}{r} (1011)_2 \\ + (11)_2 \\ \hline \end{array} \qquad \begin{array}{r} (1011)_2 \\ + (111)_2 \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

More Addition Examples

Radix-3

$$\begin{array}{r} 01110 \\ (1202)_3 \\ + (222)_3 \\ \hline (2201)_3 \end{array} \qquad \begin{array}{r} 11000 \\ (1202)_3 \\ + (1120)_3 \\ \hline (10022)_3 \end{array}$$

Binary

$$\begin{array}{r} (1011)_2 \\ + (11)_2 \\ \hline \end{array} \qquad \begin{array}{r} (1011)_2 \\ + (111)_2 \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

More Addition Examples

Radix-3

$$\begin{array}{r} 01110 \\ (1202)_3 \\ + (222)_3 \\ \hline (2201)_3 \end{array} \qquad \begin{array}{r} 11000 \\ (1202)_3 \\ + (1120)_3 \\ \hline (10022)_3 \end{array}$$

Binary

$$\begin{array}{r} 0 \\ (1011)_2 \\ + (11)_2 \\ \hline ()_2 \end{array} \qquad \begin{array}{r} (1011)_2 \\ + (111)_2 \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

More Addition Examples

Radix-3

$$\begin{array}{r} 01110 \\ (1202)_3 \\ + (222)_3 \\ \hline (2201)_3 \end{array} \qquad \begin{array}{r} 11000 \\ (1202)_3 \\ + (1120)_3 \\ \hline (10022)_3 \end{array}$$

Binary

$$\begin{array}{r} 10 \\ (1011)_2 \\ + (11)_2 \\ \hline (0)_2 \end{array} \qquad \begin{array}{r} (1011)_2 \\ + (111)_2 \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

More Addition Examples

Radix-3

$$\begin{array}{r} 01110 \\ (1202)_3 \\ + (222)_3 \\ \hline (2201)_3 \end{array} \qquad \begin{array}{r} 11000 \\ (1202)_3 \\ + (1120)_3 \\ \hline (10022)_3 \end{array}$$

Binary

$$\begin{array}{r} 110 \\ (1011)_2 \\ + (11)_2 \\ \hline (10)_2 \end{array} \qquad \begin{array}{r} (1011)_2 \\ + (111)_2 \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

More Addition Examples

Radix-3

$$\begin{array}{r} 01110 \\ (1202)_3 \\ + (222)_3 \\ \hline (2201)_3 \end{array} \qquad \begin{array}{r} 11000 \\ (1202)_3 \\ + (1120)_3 \\ \hline (10022)_3 \end{array}$$

Binary

$$\begin{array}{r} 0110 \\ (1011)_2 \\ + (11)_2 \\ \hline (110)_2 \end{array} \qquad \begin{array}{r} (1011)_2 \\ + (111)_2 \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

More Addition Examples

Radix-3

$$\begin{array}{r} 01110 \\ (1202)_3 \\ + (222)_3 \\ \hline (2201)_3 \end{array} \qquad \begin{array}{r} 11000 \\ (1202)_3 \\ + (1120)_3 \\ \hline (10022)_3 \end{array}$$

Binary

$$\begin{array}{r} 00110 \\ (1011)_2 \\ + (11)_2 \\ \hline (1110)_2 \end{array} \qquad \begin{array}{r} (1011)_2 \\ + (111)_2 \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

More Addition Examples

Radix-3

$$\begin{array}{r} 01110 \\ (1202)_3 \\ + (222)_3 \\ \hline (2201)_3 \end{array} \qquad \begin{array}{r} 11000 \\ (1202)_3 \\ + (1120)_3 \\ \hline (10022)_3 \end{array}$$

Binary

$$\begin{array}{r} 00110 \\ (1011)_2 \\ + (11)_2 \\ \hline (1110)_2 \end{array} \qquad \begin{array}{r} 0 \\ (1011)_2 \\ + (111)_2 \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

More Addition Examples

Radix-3

$$\begin{array}{r} 01110 \\ (1202)_3 \\ + (222)_3 \\ \hline (2201)_3 \end{array} \qquad \begin{array}{r} 11000 \\ (1202)_3 \\ + (1120)_3 \\ \hline (10022)_3 \end{array}$$

Binary

$$\begin{array}{r} 00110 \\ (1011)_2 \\ + (11)_2 \\ \hline (1110)_2 \end{array} \qquad \begin{array}{r} 10 \\ (1011)_2 \\ + (111)_2 \\ \hline (0)_2 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

More Addition Examples

Radix-3

$$\begin{array}{r} 01110 \\ (1202)_3 \\ + (222)_3 \\ \hline (2201)_3 \end{array} \qquad \begin{array}{r} 11000 \\ (1202)_3 \\ + (1120)_3 \\ \hline (10022)_3 \end{array}$$

Binary

$$\begin{array}{r} 00110 \\ (1011)_2 \\ + (11)_2 \\ \hline (1110)_2 \end{array} \qquad \begin{array}{r} 110 \\ (1011)_2 \\ + (111)_2 \\ \hline (10)_2 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

More Addition Examples

Radix-3

$$\begin{array}{r} 01110 \\ (1202)_3 \\ + (222)_3 \\ \hline (2201)_3 \end{array} \qquad \begin{array}{r} 11000 \\ (1202)_3 \\ + (1120)_3 \\ \hline (10022)_3 \end{array}$$

Binary

$$\begin{array}{r} 00110 \\ (1011)_2 \\ + (11)_2 \\ \hline (1110)_2 \end{array} \qquad \begin{array}{r} 1110 \\ (1011)_2 \\ + (111)_2 \\ \hline (010)_2 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

More Addition Examples

Radix-3

$$\begin{array}{r} 01110 \\ (1202)_3 \\ + (222)_3 \\ \hline (2201)_3 \end{array} \qquad \begin{array}{r} 11000 \\ (1202)_3 \\ + (1120)_3 \\ \hline (10022)_3 \end{array}$$

Binary

$$\begin{array}{r} 00110 \\ (1011)_2 \\ + (11)_2 \\ \hline (1110)_2 \end{array} \qquad \begin{array}{r} 11110 \\ (1011)_2 \\ + (111)_2 \\ \hline (0010)_2 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

More Addition Examples

Radix-3

$$\begin{array}{r} 01110 \\ (1202)_3 \\ + (222)_3 \\ \hline (2201)_3 \end{array} \qquad \begin{array}{r} 11000 \\ (1202)_3 \\ + (1120)_3 \\ \hline (10022)_3 \end{array}$$

Binary

$$\begin{array}{r} 00110 \\ (1011)_2 \\ + (11)_2 \\ \hline (1110)_2 \end{array} \qquad \begin{array}{r} 11110 \\ (1011)_2 \\ + (111)_2 \\ \hline (10010)_2 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

- If $c_n = 1$ then an additional digit appears
- The lower the radix the less we need to remember
- Binary is the **best**

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

- Subtraction is a **partial** operation on the naturals
 - ▶ There is no natural for $3 - 5$
- As expected the $-$ is taken to be the negation of $+$
- The addition tables can be used for subtraction

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Subtraction Examples

Decimal (Well known!)

$$\begin{array}{r} 5678 \\ - 1234 \\ \hline \end{array} \quad \begin{array}{r} 6331 \\ - 5678 \\ \hline \end{array}$$

Radix-12

$$\begin{array}{r} (89AB)_{12} \\ - (23A4)_{12} \\ \hline \end{array} \quad \begin{array}{r} (ABBA)_{12} \\ - (89AB)_{12} \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Subtraction Examples

Decimal (Well known!)

$$\begin{array}{r} 0 \\ 5678 \\ - 1234 \\ \hline \end{array} \quad \begin{array}{r} 6331 \\ - 5678 \\ \hline \end{array}$$

Radix-12

$$\begin{array}{r} (89AB)_{12} \\ - (23A4)_{12} \\ \hline \end{array} \quad \begin{array}{r} (ABBA)_{12} \\ - (89AB)_{12} \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Subtraction Examples

Decimal (Well known!)

$$\begin{array}{r} \\ 5678 \\ - 1234 \\ \hline 4 \end{array} \qquad \begin{array}{r} 6331 \\ - 5678 \\ \hline \end{array}$$

Radix-12

$$\begin{array}{r} (89AB)_{12} \\ - (23A4)_{12} \\ \hline \end{array} \qquad \begin{array}{r} (ABBA)_{12} \\ - (89AB)_{12} \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Subtraction Examples

Decimal (Well known!)

$$\begin{array}{r} \\ 5678 \\ - 1234 \\ \hline 44 \end{array} \qquad \begin{array}{r} 6331 \\ - 5678 \\ \hline \end{array}$$

Radix-12

$$\begin{array}{r} (89AB)_{12} \\ - (23A4)_{12} \\ \hline \end{array} \qquad \begin{array}{r} (ABBA)_{12} \\ - (89AB)_{12} \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Subtraction Examples

Decimal (Well known!)

$$\begin{array}{r} \\ 5678 \\ - 1234 \\ \hline 4444 \end{array} \quad \begin{array}{r} 6331 \\ - 5678 \\ \hline \end{array}$$

Radix-12

$$\begin{array}{r} (89AB)_{12} \\ - (23A4)_{12} \\ \hline \end{array} \quad \begin{array}{r} (ABBA)_{12} \\ - (89AB)_{12} \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Subtraction Examples

Decimal (Well known!)

$$\begin{array}{r} \\ 5678 \\ - 1234 \\ \hline 4444 \end{array} \qquad \begin{array}{r} 6331 \\ - 5678 \\ \hline \end{array}$$

Radix-12

$$\begin{array}{r} (89AB)_{12} \\ - (23A4)_{12} \\ \hline \end{array} \qquad \begin{array}{r} (ABBA)_{12} \\ - (89AB)_{12} \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Subtraction Examples

Decimal (Well known!)

$$\begin{array}{r} 000 \\ 5678 \\ - 1234 \\ \hline 4444 \end{array} \qquad \begin{array}{r} 0 \\ 6331 \\ - 5678 \\ \hline \end{array}$$

Radix-12

$$\begin{array}{r} (89AB)_{12} \\ - (23A4)_{12} \\ \hline \end{array} \qquad \begin{array}{r} (ABBA)_{12} \\ - (89AB)_{12} \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Subtraction Examples

Decimal (Well known!)

$$\begin{array}{r} \\ 5678 \\ - 1234 \\ \hline 4444 \end{array} \qquad \begin{array}{r} \\ 6331 \\ - 5678 \\ \hline 3 \end{array}$$

Radix-12

$$\begin{array}{r} (89AB)_{12} \\ - (23A4)_{12} \\ \hline \end{array} \qquad \begin{array}{r} (ABBA)_{12} \\ - (89AB)_{12} \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Subtraction Examples

Decimal (Well known!)

$$\begin{array}{r} 00000 \\ 5678 \\ - 1234 \\ \hline 4444 \end{array} \quad \begin{array}{r} 110 \\ 6331 \\ - 5678 \\ \hline 53 \end{array}$$

Radix-12

$$\begin{array}{r} (89AB)_{12} \\ - (23A4)_{12} \\ \hline \end{array} \quad \begin{array}{r} (ABBA)_{12} \\ - (89AB)_{12} \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Subtraction Examples

Decimal (Well known!)

$$\begin{array}{r} 0000 \\ 5678 \\ - 1234 \\ \hline 4444 \end{array} \qquad \begin{array}{r} 1110 \\ 6331 \\ - 5678 \\ \hline 653 \end{array}$$

Radix-12

$$\begin{array}{r} (89AB)_{12} \\ - (23A4)_{12} \\ \hline \end{array} \qquad \begin{array}{r} (ABBA)_{12} \\ - (89AB)_{12} \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Subtraction Examples

Decimal (Well known!)

$$\begin{array}{r} \\ 5678 \\ - 1234 \\ \hline 4444 \end{array} \qquad \begin{array}{r} \\ 6331 \\ - 5678 \\ \hline 653 \end{array}$$

Radix-12

$$\begin{array}{r} (89AB)_{12} \\ - (23A4)_{12} \\ \hline \end{array} \qquad \begin{array}{r} (ABBA)_{12} \\ - (89AB)_{12} \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Subtraction Examples

Decimal (Well known!)

$$\begin{array}{r} \\ 5678 \\ - 1234 \\ \hline 4444 \end{array} \qquad \begin{array}{r} \\ 6331 \\ - 5678 \\ \hline 653 \end{array}$$

Radix-12

$$\begin{array}{r} (89AB)_{12} \\ - (23A4)_{12} \\ \hline \end{array} \qquad \begin{array}{r} (ABBA)_{12} \\ - (89AB)_{12} \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Subtraction Examples

Decimal (Well known!)

$$\begin{array}{r} 00000 \\ 5678 \\ - 1234 \\ \hline 4444 \end{array} \qquad \begin{array}{r} 01110 \\ 6331 \\ - 5678 \\ \hline 653 \end{array}$$

Radix-12

$$\begin{array}{r} 0 \\ (89AB)_{12} \\ - (23A4)_{12} \\ \hline ()_{12} \end{array} \qquad \begin{array}{r} (ABBA)_{12} \\ - (89AB)_{12} \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Subtraction Examples

Decimal (Well known!)

$$\begin{array}{r} 00000 \\ 5678 \\ - 1234 \\ \hline 4444 \end{array} \quad \begin{array}{r} 01110 \\ 6331 \\ - 5678 \\ \hline 653 \end{array}$$

Radix-12

$$\begin{array}{r} 00 \\ (89AB)_{12} \\ - (23A4)_{12} \\ \hline (7)_{12} \end{array} \quad \begin{array}{r} (ABBA)_{12} \\ - (89AB)_{12} \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Subtraction Examples

Decimal (Well known!)

$$\begin{array}{r} 00000 \\ 5678 \\ - 1234 \\ \hline 4444 \end{array} \qquad \begin{array}{r} 01110 \\ 6331 \\ - 5678 \\ \hline 653 \end{array}$$

Radix-12

$$\begin{array}{r} 000 \\ (89AB)_{12} \\ - (23A4)_{12} \\ \hline (07)_{12} \end{array} \qquad \begin{array}{r} (ABBA)_{12} \\ - (89AB)_{12} \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Subtraction Examples

Decimal (Well known!)

$$\begin{array}{r} 00000 \\ 5678 \\ - 1234 \\ \hline 4444 \end{array} \quad \begin{array}{r} 01110 \\ 6331 \\ - 5678 \\ \hline 653 \end{array}$$

Radix-12

$$\begin{array}{r} 0000 \\ (89AB)_{12} \\ - (23A4)_{12} \\ \hline (607)_{12} \end{array} \quad \begin{array}{r} (ABBA)_{12} \\ - (89AB)_{12} \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Subtraction Examples

Decimal (Well known!)

$$\begin{array}{r} 00000 \\ 5678 \\ - 1234 \\ \hline 4444 \end{array} \qquad \begin{array}{r} 01110 \\ 6331 \\ - 5678 \\ \hline 653 \end{array}$$

Radix-12

$$\begin{array}{r} 00000 \\ (89AB)_{12} \\ - (23A4)_{12} \\ \hline (6607)_{12} \end{array} \qquad \begin{array}{r} (ABBA)_{12} \\ - (89AB)_{12} \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Subtraction Examples

Decimal (Well known!)

$$\begin{array}{r} 00000 \\ 5678 \\ - 1234 \\ \hline 4444 \end{array} \qquad \begin{array}{r} 01110 \\ 6331 \\ - 5678 \\ \hline 653 \end{array}$$

Radix-12

$$\begin{array}{r} 00000 \\ (89AB)_{12} \\ - (23A4)_{12} \\ \hline (6607)_{12} \end{array} \qquad \begin{array}{r} 0 \\ (ABBA)_{12} \\ - (89AB)_{12} \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Subtraction Examples

Decimal (Well known!)

$$\begin{array}{r} 00000 \\ 5678 \\ - 1234 \\ \hline 4444 \end{array} \qquad \begin{array}{r} 01110 \\ 6331 \\ - 5678 \\ \hline 653 \end{array}$$

Radix-12

$$\begin{array}{r} 00000 \\ (89AB)_{12} \\ - (23A4)_{12} \\ \hline (6607)_{12} \end{array} \qquad \begin{array}{r} 10 \\ (ABBA)_{12} \\ - (89AB)_{12} \\ \hline (B)_{12} \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Subtraction Examples

Decimal (Well known!)

$$\begin{array}{r} 00000 \\ 5678 \\ - 1234 \\ \hline 4444 \end{array} \qquad \begin{array}{r} 01110 \\ 6331 \\ - 5678 \\ \hline 653 \end{array}$$

Radix-12

$$\begin{array}{r} 00000 \\ (89AB)_{12} \\ - (23A4)_{12} \\ \hline (6607)_{12} \end{array} \qquad \begin{array}{r} 010 \\ (ABBA)_{12} \\ - (89AB)_{12} \\ \hline (0B)_{12} \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Subtraction Examples

Decimal (Well known!)

$$\begin{array}{r} 00000 \\ 5678 \\ - 1234 \\ \hline 4444 \end{array} \quad \begin{array}{r} 01110 \\ 6331 \\ - 5678 \\ \hline 653 \end{array}$$

Radix-12

$$\begin{array}{r} 00000 \\ (89AB)_{12} \\ - (23A4)_{12} \\ \hline (6607)_{12} \end{array} \quad \begin{array}{r} 0010 \\ (ABBA)_{12} \\ - (89AB)_{12} \\ \hline (20B)_{12} \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Subtraction Examples

Decimal (Well known!)

$$\begin{array}{r} 00000 \\ 5678 \\ - 1234 \\ \hline 4444 \end{array} \quad \begin{array}{r} 01110 \\ 6331 \\ - 5678 \\ \hline 653 \end{array}$$

Radix-12

$$\begin{array}{r} 00000 \\ (89AB)_{12} \\ - (23A4)_{12} \\ \hline (6607)_{12} \end{array} \quad \begin{array}{r} 00010 \\ (ABBA)_{12} \\ - (89AB)_{12} \\ \hline (220B)_{12} \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

More Subtraction Examples

Radix-3

$$\begin{array}{r} (1202)_3 \\ - (222)_3 \\ \hline \end{array} \quad \begin{array}{r} (1202)_3 \\ - (1120)_3 \\ \hline \end{array}$$

Binary

$$\begin{array}{r} (1011)_2 \\ - (11)_2 \\ \hline \end{array} \quad \begin{array}{r} (1011)_2 \\ - (111)_2 \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

More Subtraction Examples

Radix-3

$$\begin{array}{r} 0 \\ (1202)_3 \\ - (222)_3 \\ \hline (\quad)_3 \end{array} \quad \begin{array}{r} (1202)_3 \\ - (1120)_3 \\ \hline \end{array}$$

Binary

$$\begin{array}{r} (1011)_2 \\ - (11)_2 \\ \hline \end{array} \quad \begin{array}{r} (1011)_2 \\ - (111)_2 \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

More Subtraction Examples

Radix-3

$$\begin{array}{r} \\ (1202)_3 \\ - (222)_3 \\ \hline (0)_3 \end{array} \quad \begin{array}{r} (1202)_3 \\ - (1120)_3 \\ \hline \end{array}$$

Binary

$$\begin{array}{r} (1011)_2 \\ - (11)_2 \\ \hline \end{array} \quad \begin{array}{r} (1011)_2 \\ - (111)_2 \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

More Subtraction Examples

Radix-3

$$\begin{array}{r} 100 \\ (1202)_3 \\ - (222)_3 \\ \hline (10)_3 \end{array} \quad \begin{array}{r} (1202)_3 \\ - (1120)_3 \\ \hline \end{array}$$

Binary

$$\begin{array}{r} (1011)_2 \\ - (11)_2 \\ \hline \end{array} \quad \begin{array}{r} (1011)_2 \\ - (111)_2 \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

More Subtraction Examples

Radix-3

$$\begin{array}{r} 1100 \\ (1202)_3 \\ - (222)_3 \\ \hline (210)_3 \end{array} \quad \begin{array}{r} (1202)_3 \\ - (1120)_3 \\ \hline \end{array}$$

Binary

$$\begin{array}{r} (1011)_2 \\ - (11)_2 \\ \hline \end{array} \quad \begin{array}{r} (1011)_2 \\ - (111)_2 \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

More Subtraction Examples

Radix-3

$$\begin{array}{r} 01100 \\ (1202)_3 \\ - (222)_3 \\ \hline (210)_3 \end{array} \quad \begin{array}{r} (1202)_3 \\ - (1120)_3 \\ \hline \end{array}$$

Binary

$$\begin{array}{r} (1011)_2 \\ - (11)_2 \\ \hline \end{array} \quad \begin{array}{r} (1011)_2 \\ - (111)_2 \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

More Subtraction Examples

Radix-3

$$\begin{array}{r} 01100 \\ (1202)_3 \\ - (222)_3 \\ \hline (210)_3 \end{array} \qquad \begin{array}{r} 0 \\ (1202)_3 \\ - (1120)_3 \\ \hline \end{array}$$

Binary

$$\begin{array}{r} (1011)_2 \\ - (11)_2 \\ \hline \end{array} \qquad \begin{array}{r} (1011)_2 \\ - (111)_2 \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

More Subtraction Examples

Radix-3

$$\begin{array}{r} 01100 \\ (1202)_3 \\ - (222)_3 \\ \hline (210)_3 \end{array} \quad \begin{array}{r} 00 \\ (1202)_3 \\ - (1120)_3 \\ \hline (2)_3 \end{array}$$

Binary

$$\begin{array}{r} (1011)_2 \\ - (11)_2 \\ \hline \end{array} \quad \begin{array}{r} (1011)_2 \\ - (111)_2 \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

More Subtraction Examples

Radix-3

$$\begin{array}{r} 01100 \\ (1202)_3 \\ - (222)_3 \\ \hline (210)_3 \end{array} \qquad \begin{array}{r} 100 \\ (1202)_3 \\ - (1120)_3 \\ \hline (12)_3 \end{array}$$

Binary

$$\begin{array}{r} (1011)_2 \\ - (11)_2 \\ \hline \end{array} \qquad \begin{array}{r} (1011)_2 \\ - (111)_2 \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

More Subtraction Examples

Radix-3

$$\begin{array}{r} 01100 \\ (1202)_3 \\ - (222)_3 \\ \hline (210)_3 \end{array} \quad \begin{array}{r} 0100 \\ (1202)_3 \\ - (1120)_3 \\ \hline (012)_3 \end{array}$$

Binary

$$\begin{array}{r} (1011)_2 \\ - (11)_2 \\ \hline \end{array} \quad \begin{array}{r} (1011)_2 \\ - (111)_2 \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

More Subtraction Examples

Radix-3

$$\begin{array}{r} 01100 \\ (1202)_3 \\ - (222)_3 \\ \hline (210)_3 \end{array} \qquad \begin{array}{r} 00100 \\ (1202)_3 \\ - (1120)_3 \\ \hline (012)_3 \end{array}$$

Binary

$$\begin{array}{r} (1011)_2 \\ - (11)_2 \\ \hline \end{array} \qquad \begin{array}{r} (1011)_2 \\ - (111)_2 \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

More Subtraction Examples

Radix-3

$$\begin{array}{r} 01100 \\ (1202)_3 \\ - (222)_3 \\ \hline (210)_3 \end{array} \qquad \begin{array}{r} 00100 \\ (1202)_3 \\ - (1120)_3 \\ \hline (12)_3 \end{array}$$

Binary

$$\begin{array}{r} (1011)_2 \\ - (11)_2 \\ \hline \end{array} \qquad \begin{array}{r} (1011)_2 \\ - (111)_2 \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

More Subtraction Examples

Radix-3

$$\begin{array}{r} 01100 \\ (1202)_3 \\ - (222)_3 \\ \hline (210)_3 \end{array} \qquad \begin{array}{r} 00100 \\ (1202)_3 \\ - (1120)_3 \\ \hline (12)_3 \end{array}$$

Binary

$$\begin{array}{r} 0 \\ (1011)_2 \\ - (11)_2 \\ \hline ()_2 \end{array} \qquad \begin{array}{r} (1011)_2 \\ - (111)_2 \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

More Subtraction Examples

Radix-3

$$\begin{array}{r} 01100 \\ (1202)_3 \\ - (222)_3 \\ \hline (210)_3 \end{array} \qquad \begin{array}{r} 00100 \\ (1202)_3 \\ - (1120)_3 \\ \hline (12)_3 \end{array}$$

Binary

$$\begin{array}{r} 00 \\ (1011)_2 \\ - (11)_2 \\ \hline (0)_2 \end{array} \qquad \begin{array}{r} (1011)_2 \\ - (111)_2 \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

More Subtraction Examples

Radix-3

$$\begin{array}{r} 01100 \\ (1202)_3 \\ - (222)_3 \\ \hline (210)_3 \end{array} \qquad \begin{array}{r} 00100 \\ (1202)_3 \\ - (1120)_3 \\ \hline (12)_3 \end{array}$$

Binary

$$\begin{array}{r} 000 \\ (1011)_2 \\ - (11)_2 \\ \hline (00)_2 \end{array} \qquad \begin{array}{r} (1011)_2 \\ - (111)_2 \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

More Subtraction Examples

Radix-3

$$\begin{array}{r} 01100 \\ (1202)_3 \\ - (222)_3 \\ \hline (210)_3 \end{array} \qquad \begin{array}{r} 00100 \\ (1202)_3 \\ - (1120)_3 \\ \hline (12)_3 \end{array}$$

Binary

$$\begin{array}{r} 0000 \\ (1011)_2 \\ - (11)_2 \\ \hline (000)_2 \end{array} \qquad \begin{array}{r} (1011)_2 \\ - (111)_2 \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

More Subtraction Examples

Radix-3

$$\begin{array}{r} 01100 \\ (1202)_3 \\ - (222)_3 \\ \hline (210)_3 \end{array} \qquad \begin{array}{r} 00100 \\ (1202)_3 \\ - (1120)_3 \\ \hline (12)_3 \end{array}$$

Binary

$$\begin{array}{r} 00000 \\ (1011)_2 \\ - (11)_2 \\ \hline (1000)_2 \end{array} \qquad \begin{array}{r} (1011)_2 \\ - (111)_2 \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

More Subtraction Examples

Radix-3

$$\begin{array}{r} 01100 \\ (1202)_3 \\ - (222)_3 \\ \hline (210)_3 \end{array} \qquad \begin{array}{r} 00100 \\ (1202)_3 \\ - (1120)_3 \\ \hline (12)_3 \end{array}$$

Binary

$$\begin{array}{r} 00000 \\ (1011)_2 \\ - (11)_2 \\ \hline (1000)_2 \end{array} \qquad \begin{array}{r} 0 \\ (1011)_2 \\ - (111)_2 \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

More Subtraction Examples

Radix-3

$$\begin{array}{r} 01100 \\ (1202)_3 \\ - (222)_3 \\ \hline (210)_3 \end{array} \qquad \begin{array}{r} 00100 \\ (1202)_3 \\ - (1120)_3 \\ \hline (12)_3 \end{array}$$

Binary

$$\begin{array}{r} 00000 \\ (1011)_2 \\ - (11)_2 \\ \hline (1000)_2 \end{array} \qquad \begin{array}{r} 00 \\ (1011)_2 \\ - (111)_2 \\ \hline (0)_2 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

More Subtraction Examples

Radix-3

$$\begin{array}{r} 01100 \\ (1202)_3 \\ - (222)_3 \\ \hline (210)_3 \end{array} \qquad \begin{array}{r} 00100 \\ (1202)_3 \\ - (1120)_3 \\ \hline (12)_3 \end{array}$$

Binary

$$\begin{array}{r} 00000 \\ (1011)_2 \\ - (11)_2 \\ \hline (1000)_2 \end{array} \qquad \begin{array}{r} 000 \\ (1011)_2 \\ - (111)_2 \\ \hline (00)_2 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

More Subtraction Examples

Radix-3

$$\begin{array}{r} 01100 \\ (1202)_3 \\ - (222)_3 \\ \hline (210)_3 \end{array} \quad \begin{array}{r} 00100 \\ (1202)_3 \\ - (1120)_3 \\ \hline (12)_3 \end{array}$$

Binary

$$\begin{array}{r} 00000 \\ (1011)_2 \\ - (11)_2 \\ \hline (1000)_2 \end{array} \quad \begin{array}{r} 1000 \\ (1011)_2 \\ - (111)_2 \\ \hline (100)_2 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

More Subtraction Examples

Radix-3

$$\begin{array}{r} 01100 \\ (1202)_3 \\ - (222)_3 \\ \hline (210)_3 \end{array} \qquad \begin{array}{r} 00100 \\ (1202)_3 \\ - (1120)_3 \\ \hline (12)_3 \end{array}$$

Binary

$$\begin{array}{r} 00000 \\ (1011)_2 \\ - (11)_2 \\ \hline (1000)_2 \end{array} \qquad \begin{array}{r} 01000 \\ (1011)_2 \\ - (111)_2 \\ \hline (100)_2 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Definition (Semi long multiplication)

$$\langle d_{n-1}^0 \cdots d_0^0 \rangle \times_b \langle d \rangle = \langle c_n e_{n-1} \cdots e_0 \rangle,$$

where

$$c_0 = 0,$$

$$d_i = d_i^0 \times d + c_i,$$

$$c_{i+1} = \lfloor d_i / b \rfloor$$

and

$$e_i = d_i - c_{i+1} \times b,$$

for each $i < n$.

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

The Semi Long Multiplication Lemma **extra**

Numbers and
Notations

©C.M.

Lemma

Assume $x = (d_{n-1}^0 \cdots d_0^0)_b$. Then

$$x \times (d)_b = (\langle d_{n-1}^0 \cdots d_0^0 \rangle \times_b \langle d \rangle)_b.$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Proof of the 1st Multiplication Lemma (1) **extra**

Begin with the obvious:

$$\begin{aligned}x \times (d)_b &= \left(\sum_{i=0}^{n-1} d_i^0 \times b^i \right) \times d = \\&= \sum_{i=0}^{n-1} (d_i^0 \times d) \times b^i.\end{aligned}$$

This does not yield a b -radix notation since we might have i 's for which $d_i^0 \times d \geq b$.

Using the c_i and e_i from the definition of $\times_b$ we will prove by induction that for each $k \leq n$,

$$m + n = \sum_{i=0}^{k-1} e_i \times b^i + c_k \times b^k + \sum_{i=k}^{n-1} (d_i^0 \times d) \times b^i.$$

The point is that $e_i, c_i < b$, thus when $k = n$ the theorem is proved.

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Proof of the 1st Multi. Lemma (2) **extra**

The case $k = 0$ is degenerate. Assuming $k < n$ we prove the case $k + 1$.

$$\begin{aligned} \sum_{i=0}^{n-1} (d_i^0 \times d) \times b^i &= \\ &= \sum_{i=0}^{k-1} e_i \times b^i + c_k \times b^k + \sum_{i=k}^{n-1} (d_i^0 \times d) \times b^i = \\ &= \sum_{i=0}^{k-1} e_i \times b^i + c_k \times b^k + (d_k^0 \times d) \times b^k - \\ &\quad c_{k+1} \times b^{k+1} + c_{k+1} \times b^{k+1} + \\ &\quad \sum_{i=k+1}^{n-1} (d_i^0 \times d) \times b^i = \end{aligned}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Proof of the 1st Multi. Lemma (3) extra

$$\begin{aligned} &= \sum_{i=0}^{k-1} e_i \times b^i + (d_k^0 \times d + c_k - c_{k+1} \times b) \times b^k \\ &\quad + c_{k+1} \times b^{k+1} + \sum_{i=k+1}^{n-1} (d_i^0 \times d) \times b^i = \\ &= \sum_{i=0}^{k-1} e_i \times b^i + e_k \times b^k + c_{k+1} \times b^{k+1} + \\ &\quad \sum_{i=k+1}^{n-1} (d_i^0 \times d) \times b^i = \\ &= \sum_{i=0}^k e_i \times b^i + c_{k+1} \times b^{k+1} + \sum_{i=k+1}^{n-1} (d_i^0 \times d) \times b^i. \quad \square \end{aligned}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

The 'Meaning' of the 1st Multi. Lemma **extra**

Knowing the multiplication table for digits is enough

Multiplication of a number, no matter the number digits in its representation, by a digit, can be calculated using the 1st multiplication lemma

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Lemma

Fix $x = (d_{n_0-1}^0 \cdots d_0^0)_b$ and $y = (d_{n_1-1}^1 \cdots d_0^1)_b$. Then there is c^j and $e_0^j, \dots, e_{n_0-1}^j$ for each $j < n_1$ such that

$$x \times y = \sum_{j=0}^{n_1-1} (c^j e_{n_0-1}^j \cdots e_0^j)_b \times b^j.$$

- Note that multiplying by b^j means adding j zeros to the right of the representation
- Thus long multiplication is reduced to long addition, for which we have a lemma

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Proof of the Multiplication Lemma (1) extra

$$\begin{aligned}x \times y &= (d_{n_0-1}^0 \cdots d_0^0)_b \times (d_{n_1-1}^1 \cdots d_0^1)_b = \\&= \sum_{j=0}^{n_1-1} (d_{n_0-1}^0 \cdots d_0^0)_b \times d_i^1 \times b^i.\end{aligned}$$

By the 1st multiplication lemma there is $c^j, e_{n_0-1}^j, \dots, e_0^j$ such that

$$(d_{n_0-1}^0 \cdots d_0^0)_b \times d_j^1 = (c^j e_{n_0-1}^j \cdots e_0^j)_b.$$

Thus

$$x \times y = \sum_{j=0}^{n_1-1} (c^j e_{n_0-1}^j \cdots e_0^j)_b \times b^j.$$

We are done.

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Multiplication Tables (radix-3 and binary)

Radix-3

×	0	1	2
0	0	0	0
1	0	1	2
2	0	2	11

Binary

×	0	1
0	0	0
1	0	1

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Multiplication Table (Radix-12)

×	0	1	2	3	4	5	6	7	8	9	A	B
0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	A	B
2	0	2	4	6	8	A	10	12	14	16	18	1A
3	0	3	6	9	10	13	16	19	20	23	26	29
4	0	4	8	10	14	18	20	24	28	30	34	38
5	0	5	A	13	18	21	26	2B	34	39	42	47
6	0	6	10	16	20	26	30	36	40	46	50	56
7	0	7	12	19	24	2B	36	41	48	53	5A	65
8	0	8	14	20	28	34	40	48	54	60	68	74
9	0	9	16	23	30	39	46	53	60	69	76	83
A	0	A	18	26	34	42	50	5A	68	76	84	92
B	0	B	1A	29	38	47	56	65	74	83	92	A1

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Decimal

$$\begin{array}{r} 12345 \\ 789 \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Decimal

$$\begin{array}{r} 4 \\ 12345 \\ 789 \\ \hline 5 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Decimal

$$\begin{array}{r} 44 \\ 12345 \\ \times 789 \\ \hline 05 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Decimal

$$\begin{array}{r} 344 \\ 12345 \\ 789 \\ \hline 105 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Decimal

$$\begin{array}{r} 2344 \\ 12345 \\ 789 \\ \hline 1105 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Decimal

$$\begin{array}{r} 12344 \\ 12345 \\ 789 \\ \hline 11105 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Decimal

$$\begin{array}{r} 2 4 5 \\ 12345 \\ 789 \\ \hline 111105 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Decimal

$$\begin{array}{r} 12345 \\ 789 \\ \hline 111105 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Decimal

$$\begin{array}{r} 12345 \\ 789 \\ \hline 111105 \\ 0 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Decimal

$$\begin{array}{r} 4 \\ 12345 \\ \times 789 \\ \hline 111105 \\ 00 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Decimal

$$\begin{array}{r} 34 \\ 12345 \\ \times 789 \\ \hline 111105 \\ 600 \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Decimal

$$\begin{array}{r} 234 \\ 12345 \\ 789 \\ \hline 111105 \\ 7600 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Decimal

$$\begin{array}{r} 1234 \\ 12345 \\ 789 \\ \hline 111105 \\ 87600 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Decimal

$$\begin{array}{r} 1 3 5 \\ 789 \\ \hline 111105 \\ 987600 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Decimal

$$\begin{array}{r} \\ \\ \\ \\ \hline \\ \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Decimal

$$\begin{array}{r} 12345 \\ 789 \\ \hline 111105 \\ 987600 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Decimal

$$\begin{array}{r} 12345 \\ \times 789 \\ \hline 111105 \\ 987600 \\ 0 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Decimal

$$\begin{array}{r} 12345 \\ 789 \\ \hline 111105 \\ 987600 \\ 00 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Decimal

$$\begin{array}{r} 3 \\ 12345 \\ \times 789 \\ \hline 111105 \\ 987600 \\ 500 \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Decimal

$$\begin{array}{r} 33 \\ 12345 \\ 789 \\ \hline 111105 \\ 987600 \\ 1500 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Decimal

$$\begin{array}{r} 2\ 3\ 3 \\ 12345 \\ 789 \\ \hline 111105 \\ 987600 \\ 41500 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Decimal

$$\begin{array}{r} 1233 \\ 12345 \\ 789 \\ \hline 111105 \\ 987600 \\ 641500 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Decimal

$$\begin{array}{r} 0 2 3 \\ 12345 \\ 789 \\ \hline 111105 \\ 987600 \\ 8641500 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Decimal

$$\begin{array}{r} 0 2 3 \\ 12345 \\ 789 \\ \hline 111105 \\ 987600 \\ 8641500 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Decimal

$$\begin{array}{r} 12345 \\ 789 \\ \hline 111105 \\ 987600 \\ 8641500 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Decimal

$$\begin{array}{r} 12345 \\ \times 789 \\ \hline 111105 \\ 987600 \\ 8641500 \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Decimal

$$\begin{array}{r} 12345 \\ 789 \\ \hline 111105 \\ 987600 \\ 8641500 \\ \hline 5 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Decimal

$$\begin{array}{r} 12345 \\ 789 \\ \hline 111105 \\ 987600 \\ 8641500 \\ \hline 05 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Decimal

$$\begin{array}{r} 12345 \\ 789 \\ \hline 111105 \\ 987600 \\ 8641500 \\ \hline 205 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Decimal

$$\begin{array}{r} 12345 \\ \times 789 \\ \hline 111105 \\ 987600 \\ 8641500 \\ \hline 0205 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Decimal

$$\begin{array}{r} 12345 \\ 789 \\ \hline 111105 \\ 987600 \\ 8641500 \\ \hline 40205 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Decimal

$$\begin{array}{r} 12345 \\ 789 \\ \hline 111105 \\ 987600 \\ 8641500 \\ \hline 740205 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Decimal

$$\begin{array}{r} 12345 \\ \times 789 \\ \hline 111105 \\ 987600 \\ 8641500 \\ \hline 9740205 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-12

$$\begin{array}{r} 12AB3 \\ \times A1B \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-12

$$\begin{array}{r} 2 \\ 12AB3 \\ \times A1B \\ \hline 9 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-12

$$\begin{array}{r} \text{A } 2 \\ 12\text{AB}3 \\ \text{A1B} \\ \hline 39 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-12

$$\begin{array}{r} \text{A A 2} \\ 12\text{AB3} \\ \times \text{A1B} \\ \hline 039 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-12

2 A A 2

12AB3

A1B

8039

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-12

$$\begin{array}{r} 12AB3 \\ \times A1B \\ \hline 18039 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-12

$$\begin{array}{r} 12AB3 \\ \times A1B \\ \hline 118039 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-12

$$\begin{array}{r} 12AB3 \\ \times A1B \\ \hline 118039 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-12

$$\begin{array}{r} 12AB3 \\ \times A1B \\ \hline 118039 \\ 0 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-12

$$\begin{array}{r} 0 \\ 12AB3 \\ \times A1B \\ \hline 118039 \\ 30 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-12

$$\begin{array}{r} \\ 12AB3 \\ \times A1B \\ \hline 118039 \\ B30 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-12

$$\begin{array}{r} \\ 12AB3 \\ \times A1B \\ \hline 118039 \\ AB30 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-12

$$\begin{array}{r} \\ 12AB3 \\ A1B \\ \hline 118039 \\ 2AB30 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-12

$$\begin{array}{r} \\ 12AB3 \\ A1B \\ \hline 118039 \\ 12AB30 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-12

$$\begin{array}{r} \\ 12AB3 \\ A1B \\ \hline 118039 \\ 12AB30 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-12

$$\begin{array}{r} 12AB3 \\ \times A1B \\ \hline 118039 \\ 12AB30 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-12

$$\begin{array}{r} 12AB3 \\ \times A1B \\ \hline 118039 \\ 12AB30 \\ 0 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-12

$$\begin{array}{r} 12AB3 \\ \times A1B \\ \hline 118039 \\ 12AB30 \\ 00 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-12

$$\begin{array}{r} ^2 \\ 12AB3 \\ \times A1B \\ \hline 118039 \\ 12AB30 \\ \hline 600 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-12

$$\begin{array}{r} 9\ 2 \\ 12AB3 \\ \times A1B \\ \hline 118039 \\ 12AB30 \\ 4600 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-12

$$\begin{array}{r} 992 \\ 12AB3 \\ A1B \\ \hline 118039 \\ 12AB30 \\ \hline 14600 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-12

$$\begin{array}{r} 2992 \\ 12AB3 \\ \times A1B \\ \hline 118039 \\ 12AB30 \\ 514600 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-12

$$\begin{array}{r} 12992 \\ 12AB3 \\ \times A1B \\ \hline 118039 \\ 12AB30 \\ 0514600 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-12

$$\begin{array}{r} 12AB3 \\ \times A1B \\ \hline 118039 \\ 12AB30 \\ 10514600 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-12

$$\begin{array}{r} 12AB3 \\ \times A1B \\ \hline 118039 \\ 12AB30 \\ 10514600 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-12

$$\begin{array}{r} 12AB3 \\ \times A1B \\ \hline 118039 \\ 12AB30 \\ \hline 10514600 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-12

$$\begin{array}{r} 12AB3 \\ \times A1B \\ \hline 118039 \\ 12AB30 \\ \hline 10514600 \\ \hline 9 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-12

$$\begin{array}{r} 12AB3 \\ \times A1B \\ \hline 118039 \\ 12AB30 \\ \hline 10514600 \\ \hline 69 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-12

$$\begin{array}{r} 12AB3 \\ \times A1B \\ \hline 118039 \\ 12AB30 \\ \hline 10514600 \\ \hline 569 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-12

$$\begin{array}{r} 12AB3 \\ \times A1B \\ \hline 118039 \\ 12AB30 \\ \hline 10514600 \\ \hline B569 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-12

$$\begin{array}{r} 12AB3 \\ \times A1B \\ \hline 118039 \\ 12AB30 \\ 10514600 \\ \hline 5B569 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-12

$$\begin{array}{r} 12AB3 \\ \times A1B \\ \hline 118039 \\ 12AB30 \\ \hline 10514600 \\ \hline 75B569 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-12

$$\begin{array}{r} 12AB3 \\ \times A1B \\ \hline 118039 \\ 12AB30 \\ \hline 10514600 \\ \hline 075B569 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-12

$$\begin{array}{r} 12AB3 \\ \times A1B \\ \hline 118039 \\ 12AB30 \\ \hline 10514600 \\ \hline 1075B569 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-3

$$\begin{array}{r} 12012 \\ 221 \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-3

$$\begin{array}{r} 0 \\ 12012 \\ 221 \\ \hline 2 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-3

$$\begin{array}{r} 00 \\ 12012 \\ 221 \\ \hline 12 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-3

$$\begin{array}{r} \\ 12012 \\ 221 \\ \hline 012 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-3

$$\begin{array}{r} 0000 \\ 12012 \\ 221 \\ \hline 2012 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-3

$$\begin{array}{r} 00000 \\ 12012 \\ 221 \\ \hline 12012 \end{array}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Multiplication Example

Radix-3

$$\begin{array}{r} 00000 \\ 12012 \\ 221 \\ \hline 12012 \end{array}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Multiplication Example

Radix-3

$$\begin{array}{r} 12012 \\ 221 \\ \hline 12012 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-3

$$\begin{array}{r} 12012 \\ 221 \\ \hline 12012 \\ 0 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-3

$$\begin{array}{r} 1 \\ 12012 \\ 221 \\ \hline 12012 \\ 10 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-3

$$\begin{array}{r} 1 \\ 12012 \\ 221 \\ \hline 12012 \\ 010 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-3

$$\begin{array}{r} 11 \\ 12012 \\ 221 \\ \hline 12012 \\ 1010 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-3

$$\begin{array}{r} 1011 \\ 12012 \\ 221 \\ \hline 12012 \\ 11010 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-3

$$\begin{array}{r} 11011 \\ 12012 \\ 221 \\ \hline 12012 \\ 011010 \end{array}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Multiplication Example

Radix-3

$$\begin{array}{r} 11011 \\ 12012 \\ \times 221 \\ \hline 12012 \\ 1011010 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-3

$$\begin{array}{r} 12012 \\ 221 \\ \hline 12012 \\ 1011010 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-3

$$\begin{array}{r} 12012 \\ \times 221 \\ \hline 12012 \\ 1011010 \\ 0 \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-3

$$\begin{array}{r} 12012 \\ 221 \\ \hline 12012 \\ 1011010 \\ 00 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-3

$$\begin{array}{r} 1 \\ 12012 \\ 221 \\ \hline 12012 \\ 1011010 \\ 100 \end{array}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Multiplication Example

Radix-3

$$\begin{array}{r} 11 \\ 12012 \\ \times 221 \\ \hline 12012 \\ 1011010 \\ 0100 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-3

$$\begin{array}{r} 1 1 \\ 12012 \\ 221 \\ \hline 12012 \\ 1011010 \\ 10100 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-3

$$\begin{array}{r} 1011 \\ 12012 \\ 221 \\ \hline 12012 \\ 1011010 \\ 110100 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-3

$$\begin{array}{r} 11011 \\ 12012 \\ 221 \\ \hline 12012 \\ 1011010 \\ 0110100 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-3

$$\begin{array}{r} 11011 \\ 12012 \\ 221 \\ \hline 12012 \\ 1011010 \\ 10110100 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-3

$$\begin{array}{r} 12012 \\ 221 \\ \hline 12012 \\ 1011010 \\ 10110100 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-3

$$\begin{array}{r} 12012 \\ 221 \\ \hline 12012 \\ 1011010 \\ 10110100 \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-3

$$\begin{array}{r} 12012 \\ 221 \\ \hline 12012 \\ 1011010 \\ 10110100 \\ \hline 2 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-3

$$\begin{array}{r} 12012 \\ 221 \\ \hline 12012 \\ 1011010 \\ 10110100 \\ \hline 22 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-3

$$\begin{array}{r} 12012 \\ 221 \\ \hline 12012 \\ 1011010 \\ 10110100 \\ \hline 122 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-3

$$\begin{array}{r} 12012 \\ 221 \\ \hline 12012 \\ 1011010 \\ 10110100 \\ \hline 0122 \end{array}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Multiplication Example

Radix-3

$$\begin{array}{r} 12012 \\ 221 \\ \hline 12012 \\ 1011010 \\ 10110100 \\ \hline 10122 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-3

$$\begin{array}{r} 12012 \\ 221 \\ \hline 12012 \\ 1011010 \\ 10110100 \\ \hline 210122 \end{array}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Multiplication Example

Radix-3

$$\begin{array}{r} 12012 \\ 221 \\ \hline 12012 \\ 1011010 \\ 10110100 \\ \hline 1210122 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Radix-3

$$\begin{array}{r} 12012 \\ 221 \\ \hline 12012 \\ 1011010 \\ 10110100 \\ \hline 11210122 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Binary

11011

101

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Binary

$$\begin{array}{r} 0 \\ 11011 \\ 101 \\ \hline 1 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Binary

$$\begin{array}{r} 00 \\ 11011 \\ 101 \\ \hline 11 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Binary

$$\begin{array}{r} 000 \\ 11011 \\ 101 \\ \hline 011 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Binary

$$\begin{array}{r} 0000 \\ 11011 \\ 101 \\ \hline 1011 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Binary

$$\begin{array}{r} 00000 \\ 11011 \\ 101 \\ \hline 11011 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Binary

$$\begin{array}{r} 00000 \\ 11011 \\ 101 \\ \hline 11011 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Binary

$$\begin{array}{r} 11011 \\ 101 \\ \hline 11011 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Binary

$$\begin{array}{r} 11011 \\ 101 \\ \hline 11011 \\ 0 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Binary

$$\begin{array}{r} 0 \\ 11011 \\ 101 \\ \hline 11011 \\ 00 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Binary

$$\begin{array}{r} 00 \\ 11011 \\ 101 \\ \hline 11011 \\ 000 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Binary

$$\begin{array}{r} 000 \\ 11011 \\ 101 \\ \hline 11011 \\ 0000 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Binary

$$\begin{array}{r} 0000 \\ 11011 \\ 101 \\ \hline 11011 \\ 00000 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Binary

$$\begin{array}{r} 00000 \\ 11011 \\ 101 \\ \hline 11011 \\ 000000 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Binary

$$\begin{array}{r} 00000 \\ 11011 \\ 101 \\ \hline 11011 \\ 000000 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Binary

$$\begin{array}{r} 11011 \\ 101 \\ \hline 11011 \\ 000000 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Binary

$$\begin{array}{r} 11011 \\ 101 \\ \hline 11011 \\ 000000 \\ 0 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Binary

$$\begin{array}{r} 11011 \\ 101 \\ \hline 11011 \\ 000000 \\ 00 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Binary

$$\begin{array}{r} 0 \\ 11011 \\ 101 \\ \hline 11011 \\ 000000 \\ 100 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Binary

$$\begin{array}{r} 00 \\ 11011 \\ 101 \\ \hline 11011 \\ 000000 \\ 1100 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Binary

$$\begin{array}{r} 000 \\ 11011 \\ 101 \\ \hline 11011 \\ 000000 \\ 01100 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Binary

$$\begin{array}{r} 0000 \\ 11011 \\ \times 101 \\ \hline 11011 \\ 000000 \\ 101100 \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Binary

$$\begin{array}{r} \\ 11011 \\ 101 \\ \hline 11011 \\ 000000 \\ \hline 1101100 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Binary

$$\begin{array}{r} \\ 11011 \\ 101 \\ \hline 11011 \\ 000000 \\ \hline 1101100 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Binary

$$\begin{array}{r} 11011 \\ 101 \\ \hline 11011 \\ 000000 \\ 1101100 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Binary

$$\begin{array}{r} 11011 \\ 101 \\ \hline 11011 \\ 00000 \\ 1101100 \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Binary

$$\begin{array}{r} 11011 \\ 101 \\ \hline 11011 \\ 000000 \\ 1101100 \\ \hline 1 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Binary

$$\begin{array}{r} 11011 \\ 101 \\ \hline 11011 \\ 00000 \\ 1101100 \\ \hline 11 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Binary

$$\begin{array}{r} 11011 \\ 101 \\ \hline 11011 \\ 00000 \\ 1101100 \\ \hline 111 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Binary

$$\begin{array}{r} 11011 \\ 101 \\ \hline 11011 \\ 000000 \\ 1101100 \\ \hline 0111 \end{array}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Multiplication Example

Binary

$$\begin{array}{r} 11011 \\ 101 \\ \hline 11011 \\ 000000 \\ 1101100 \\ \hline 00111 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Binary

$$\begin{array}{r} 11011 \\ 101 \\ \hline 11011 \\ 000000 \\ 1101100 \\ \hline 000111 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Binary

$$\begin{array}{r} 11011 \\ 101 \\ \hline 11011 \\ 000000 \\ 1101100 \\ \hline 0000111 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Multiplication Example

Binary

$$\begin{array}{r} 11011 \\ 101 \\ \hline 11011 \\ 00000 \\ 1101100 \\ \hline 10000111 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

- Division is a **partial** operation on the naturals
 - ▶ There is no natural for $3 \div 5$
- As expected the $\div$ is taken to be the negation of $\times$
 - ▶ There might be a remainder
- The multiplication tables can be used for division

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Division Example (Radix 10)

$$\overline{9740305} \overline{)789}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Division Example (Radix 10)

$$\begin{array}{r} 1 \\ \hline 9740305 \overline{)789} \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Division Example (Radix 10)

$$\begin{array}{r} 1 \\ \hline 9740305 \overline{)789} \\ 789 \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Division Example (Radix 10)

$$\begin{array}{r} 1 \\ \hline 9740305 \overline{)789} \\ 789 \\ \hline 185 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Division Example (Radix 10)

$$\begin{array}{r} 1 \\ \hline 9740305 \overline{)789} \\ 789 \\ \hline 1850 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Division Example (Radix 10)

$$\begin{array}{r} 12 \\ \hline 9740305 \overline{)789} \\ 789 \\ \hline 1850 \end{array}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Division Example (Radix 10)

$$\begin{array}{r} 12 \\ \hline 9740305 \overline{)789} \\ 789 \\ \hline 1850 \\ 1578 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Division Example (Radix 10)

$$\begin{array}{r} 12 \\ \hline 9740305 \overline{)789} \\ 789 \\ \hline 1850 \\ 1578 \\ \hline 272 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Division Example (Radix 10)

$$\begin{array}{r} 12 \\ \hline 9740305 \overline{)789} \\ 789 \\ \hline 1850 \\ 1578 \\ \hline 2723 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Division Example (Radix 10)

$$\begin{array}{r} 123 \\ \hline 9740305 \overline{)789} \\ 789 \\ \hline 1850 \\ 1578 \\ \hline 2723 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Division Example (Radix 10)

$$\begin{array}{r} 123 \\ \hline 9740305 \overline{)789} \\ 789 \\ \hline 1850 \\ 1578 \\ \hline 2723 \\ 2367 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Division Example (Radix 10)

$$\begin{array}{r} 123 \\ \hline 9740305 \overline{)789} \\ 789 \\ \hline 1850 \\ 1578 \\ \hline 2723 \\ 2367 \\ \hline 356 \end{array}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Division Example (Radix 10)

$$\begin{array}{r} 123 \\ \hline 9740305 \overline{)789} \\ 789 \\ \hline 1850 \\ 1578 \\ \hline 2723 \\ 2367 \\ \hline 3560 \end{array}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Division Example (Radix 10)

$$\begin{array}{r} 1234 \\ 9740305 \overline{) 789} \\ 789 \\ \hline 1850 \\ 1578 \\ \hline 2723 \\ 2367 \\ \hline 3560 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Division Example (Radix 10)

$$\begin{array}{r} 1234 \\ 9740305 \overline{) 789} \\ 789 \\ \hline 1850 \\ 1578 \\ \hline 2723 \\ 2367 \\ \hline 3560 \\ 3156 \end{array}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Division Example (Radix 10)

$$\begin{array}{r} 1234 \\ \hline 9740305 \overline{)789} \\ 789 \\ \hline 1850 \\ 1578 \\ \hline 2723 \\ 2367 \\ \hline 3560 \\ 3156 \\ \hline 404 \end{array}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Division Example (Radix 10)

$$\begin{array}{r} 1234 \\ \hline 9740305 \overline{) 789} \\ 789 \\ \hline 1850 \\ 1578 \\ \hline 2723 \\ 2367 \\ \hline 3560 \\ 3156 \\ \hline 4045 \end{array}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Division Example (Radix 10)

$$\begin{array}{r} 12345 \\ 9740305 \overline{) 789} \\ 789 \\ \hline 1850 \\ 1578 \\ \hline 2723 \\ 2367 \\ \hline 3560 \\ 3156 \\ \hline 4045 \end{array}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Division Example (Radix 10)

$$\begin{array}{r} 12345 \\ 9740305 \overline{) 789} \\ 789 \\ \hline 1850 \\ 1578 \\ \hline 2723 \\ 2367 \\ \hline 3560 \\ 3156 \\ \hline 4045 \\ 3945 \end{array}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Division Example (Radix 10)

$$\begin{array}{r} 12345 \\ 9740305 \overline{) 789} \\ 789 \\ \hline 1850 \\ 1578 \\ \hline 2723 \\ 2367 \\ \hline 3560 \\ 3156 \\ \hline 4045 \\ 3945 \\ \hline \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Division Example (Radix 10)

$$\begin{array}{r} 12345 \\ 9740305 \overline{) 789} \\ 789 \\ \hline 1850 \\ 1578 \\ \hline 2723 \\ 2367 \\ \hline 3560 \\ 3156 \\ \hline 4045 \\ 3945 \\ \hline 100 \end{array}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Division Example (Radix 10)

$$\begin{array}{r} 12345 \\ 9740305 \overline{) 789} \\ \underline{789} \\ 1850 \\ \underline{1578} \\ 2723 \\ \underline{2367} \\ 3560 \\ \underline{3156} \\ 4045 \\ \underline{3945} \\ 100 \end{array}$$

$$9740305 = 12345 \times 789 + 100$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Division Example (Radix 12)

$$\overline{1075B569} \overline{)A1B}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Division Example (Radix 12)

$$\begin{array}{r} 0 \\ \hline 1075B569 \overline{)A1B} \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Division Example (Radix 12)

$$\begin{array}{r} 0 \\ 1075B569 \overline{)A1B} \\ 0 \end{array}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Division Example (Radix 12)

$$\begin{array}{r} 0 \\ \hline 1075B569 \overline{)A1B} \\ 0 \\ \hline 107 \end{array}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Division Example (Radix 12)

$$\begin{array}{r} 0 \\ \hline 1075B569 \overline{)A1B} \\ 0 \\ \hline 1075 \end{array}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Division Example (Radix 12)

$$\begin{array}{r} 01 \\ \hline 1075B569 \overline{)A1B} \\ \underline{0} \\ 1075 \end{array}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Division Example (Radix 12)

$$\begin{array}{r} 01 \\ \hline 1075B569 \overline{)A1B} \\ \underline{0} \\ 1075 \\ \underline{A1B} \end{array}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Division Example (Radix 12)

$$\begin{array}{r} 01 \\ \hline 1075B569 \overline{)A1B} \\ \underline{0} \\ 1075 \\ \underline{A1B} \\ 256 \end{array}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Division Example (Radix 12)

$$\begin{array}{r} 01 \\ \hline 1075B569 \overline{)A1B} \\ \underline{0} \\ 1075 \\ \underline{A1B} \\ 256B \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Division Example (Radix 12)

$$\begin{array}{r} 012 \\ \hline 1075B569 \overline{)A1B} \\ \underline{0} \\ 1075 \\ \underline{A1B} \\ 256B \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Division Example (Radix 12)

$$\begin{array}{r} 012 \\ \hline 1075B569 \overline{)A1B} \\ \underline{0} \\ 1075 \\ \underline{A1B} \\ 256B \\ \underline{183A} \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Division Example (Radix 12)

$$\begin{array}{r} 012 \\ \hline 1075B569 \overline{)A1B} \\ \underline{0} \\ 1075 \\ \underline{A1B} \\ 256B \\ \underline{183A} \\ 931 \end{array}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Division Example (Radix 12)

$$\begin{array}{r} 012 \\ \hline 1075B569 \overline{)A1B} \\ \underline{0} \\ 1075 \\ \underline{A1B} \\ 256B \\ \underline{183A} \\ 9315 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Division Example (Radix 12)

$$\begin{array}{r} 012A \\ \hline 1075B569 \overline{)A1B} \\ \underline{0} \\ 1075 \\ \hline A1B \\ \hline 256B \\ \hline 183A \\ \hline 9315 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Division Example (Radix 12)

$$\begin{array}{r} 012A \\ \hline 1075B569 \overline{)A1B} \\ \underline{0} \\ 1075 \\ \hline A1B \\ \hline 256B \\ \hline 183A \\ \hline 9315 \\ \hline 8572 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Division Example (Radix 12)

$$\begin{array}{r} 012A \\ \hline 1075B569 \overline{)A1B} \\ \underline{0} \\ 1075 \\ \hline A1B \\ \hline 256B \\ \hline 183A \\ \hline 9315 \\ \hline 8572 \\ \hline 963 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Division Example (Radix 12)

$$\begin{array}{r} 012A \\ \overline{1075B569} \overline{)A1B} \\ \underline{0} \\ 1075 \\ \underline{A1B} \\ 256B \\ \underline{183A} \\ 9315 \\ \underline{8572} \\ 9636 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Division Example (Radix 12)

$$\begin{array}{r} 012AB \\ \hline 1075B569 \overline{)A1B} \\ \underline{0} \\ 1075 \\ \hline A1B \\ \hline 256B \\ \hline 183A \\ \hline 9315 \\ \hline 8572 \\ \hline 9636 \end{array}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Division Example (Radix 12)

$$\begin{array}{r} 012AB \\ \underline{1075B569} \\ 0 \\ \underline{1075} \\ A1B \\ \underline{256B} \\ 183A \\ \underline{9315} \\ 8572 \\ \underline{9636} \\ 9666 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Division Example (Radix 12)

$$\begin{array}{r} 012AB \\ \hline 1075B569 \overline{)A1B} \\ \underline{0} \\ 1075 \\ \hline A1B \\ \hline 256B \\ \hline 183A \\ \hline 9315 \\ \hline 8572 \\ \hline 9636 \\ \hline 9666 \\ \hline \end{array}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Division Example (Radix 12)

$$\begin{array}{r} 012AB \\ \hline 1075B569 \overline{)A1B} \\ \underline{0} \\ 1075 \\ \hline A1B \\ \hline 256B \\ \hline 183A \\ \hline 9315 \\ \hline 8572 \\ \hline 9636 \\ \hline 9666 \\ \hline \end{array}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Division Example (Radix 12)

$$\begin{array}{r} 012AB3 \\ \hline 1075B569 \overline{)A1B} \\ 0 \\ \hline 1075 \\ A1B \\ \hline 256B \\ 183A \\ \hline 9315 \\ 8572 \\ \hline 9636 \\ 9666 \\ \hline 0 \end{array}$$

$$(1075B569)_{12} = (12AB3)_{12} \times (A1B)_{12}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Division Example (Binary)

$$\overline{10001011} \overline{)101}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Division Example (Binary)

$$\begin{array}{r} 0 \\ 10001011 \overline{)101} \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Division Example (Binary)

$$\begin{array}{r} 0 \\ 10001011 \overline{)101} \\ 0 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Division Example (Binary)

$$\begin{array}{r} 0 \\ \hline 10001011 \overline{)101} \\ 0 \\ \hline 100 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Division Example (Binary)

$$\begin{array}{r} 0 \\ \hline 10001011 \overline{)101} \\ 0 \\ \hline 1000 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Division Example (Binary)

$$\begin{array}{r} 01 \\ 10001011 \overline{) 101} \\ \underline{0} \\ 1000 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Division Example (Binary)

$$\begin{array}{r} 01 \\ \overline{10001011} \overline{)101} \\ 0 \\ \overline{1000} \\ 101 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Division Example (Binary)

$$\begin{array}{r} 01 \\ \overline{10001011} \overline{)101} \\ 0 \\ \overline{1000} \\ 101 \\ \overline{11} \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Division Example (Binary)

$$\begin{array}{r} 01 \\ \hline 10001011 \overline{)101} \\ 0 \\ \hline 1000 \\ 101 \\ \hline 111 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Division Example (Binary)

$$\begin{array}{r} 011 \\ \overline{10001011} \overline{)101} \\ 0 \\ \overline{1000} \\ 101 \\ \overline{111} \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Division Example (Binary)

$$\begin{array}{r} 011 \\ \overline{10001011} \overline{)101} \\ 0 \\ \overline{1000} \\ 101 \\ \overline{111} \\ 101 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Division Example (Binary)

$$\begin{array}{r} 011 \\ \overline{10001011} \overline{)101} \\ 0 \\ \overline{1000} \\ 101 \\ \overline{111} \\ 101 \\ \overline{10} \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Division Example (Binary)

$$\begin{array}{r} 011 \\ \overline{10001011} \overline{)101} \\ 0 \\ \overline{1000} \\ 101 \\ \overline{111} \\ 101 \\ \overline{100} \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Division Example (Binary)

$$\begin{array}{r} 0110 \\ \overline{10001011} \overline{)101} \\ 0 \\ \overline{1000} \\ 101 \\ \overline{111} \\ 101 \\ \overline{100} \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Division Example (Binary)

$$\begin{array}{r} 0110 \\ \overline{10001011} \overline{)101} \\ 0 \\ \overline{1000} \\ 101 \\ \overline{111} \\ 101 \\ \overline{100} \\ 0 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Division Example (Binary)

$$\begin{array}{r} 0110 \\ \overline{10001011} \overline{)101} \\ \underline{0} \\ 1000 \\ \underline{101} \\ 111 \\ \underline{101} \\ 100 \\ \underline{0} \\ 100 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Division Example (Binary)

$$\begin{array}{r} 0110 \\ \overline{10001011} \overline{)101} \\ \underline{0} \\ 1000 \\ \underline{101} \\ 111 \\ \underline{101} \\ 100 \\ \underline{0} \\ 1001 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Division Example (Binary)

$$\begin{array}{r} 01101 \\ \overline{10001011} \overline{)101} \\ \underline{0} \\ 1000 \\ \underline{101} \\ 111 \\ \underline{101} \\ 100 \\ \underline{0} \\ 1001 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Division Example (Binary)

$$\begin{array}{r} 01101 \\ \overline{10001011} \overline{)101} \\ \underline{0} \\ 1000 \\ \underline{101} \\ 111 \\ \underline{101} \\ 100 \\ \underline{0} \\ 1001 \\ \underline{101} \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Division Example (Binary)

$$\begin{array}{r} 01101 \\ \hline 10001011 \overline{)101} \\ \underline{0} \\ 1000 \\ \underline{101} \\ 111 \\ \underline{101} \\ 100 \\ \underline{0} \\ 1001 \\ \underline{101} \\ 100 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Division Example (Binary)

$$\begin{array}{r} 01101 \\ \hline 10001011 \overline{) 101} \\ \underline{0} \\ 1000 \\ \underline{101} \\ 111 \\ \underline{101} \\ 100 \\ \underline{0} \\ 1001 \\ \underline{101} \\ 1001 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Division Example (Binary)

$$\begin{array}{r} 011011 \\ \hline 10001011 \overline{)101} \\ \underline{0} \\ 1000 \\ \underline{101} \\ 111 \\ \underline{101} \\ 100 \\ \underline{0} \\ 1001 \\ \underline{101} \\ 1001 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Division Example (Binary)

$$\begin{array}{r} 011011 \\ 10001011 \overline{) 101} \\ \underline{0} \\ 1000 \\ \underline{101} \\ 111 \\ \underline{101} \\ 100 \\ \underline{0} \\ 1001 \\ \underline{101} \\ 1001 \\ \underline{101} \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Division Example (Binary)

$$\begin{array}{r} 011011 \\ 10001011 \overline{) 101} \\ \underline{0} \\ 1000 \\ \underline{101} \\ 111 \\ \underline{101} \\ 100 \\ \underline{0} \\ 1001 \\ \underline{101} \\ 1001 \\ \underline{101} \\ 100 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Division Example (Binary)

$$\begin{array}{r} 011011 \\ \overline{10001011} \overline{)101} \\ 0 \\ \overline{1000} \\ 101 \\ \overline{111} \\ 101 \\ \overline{100} \\ 0 \\ \overline{1001} \\ 101 \\ \overline{1001} \\ 101 \\ \overline{100} \end{array}$$

$$(10001011)_2 = (11011)_2 \times (101)_2 + (100)_2$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

(Carmi) Lecture 1 reached here

N

- Peano
- Caveperson
- Addition
- Subtraction
- Multiplication**
- Switching radices
- Hardware

Z

- Complement
- Hardware

Q

- Examples
- Hardware
- Fixed point
- BCD
- Floating-point
- Examples

Switching Radices

Luckily we have the '+' and '×' at this point

N

- Peano
- Caveperson
- Addition
- Subtraction
- Multiplication
- Switching radices
- Hardware

Z

- Complement
- Hardware

Q

- Examples
- Hardware
- Fixed point
- BCD
- Floating-point
- Examples

Example: Radix-3 to Radix-4 (1)

Question

$$(1022)_3 = (?)_4$$

Solution 1 (Fluency with radix-10 is assumed)

Split the work into two steps:

$$(1022)_3 = x \text{ then } x = (?)_4$$

$$\begin{array}{l} \text{Convert to decimal} \\ x = 1 \times 3^3 + 0 \times 3^2 + \\ \quad 2 \times 3 + 2 \times 1 = \\ = 35 \end{array}$$

$$\begin{array}{l} \text{Convert to base 4} \\ 35 = 8 \times 4 + 3 \\ 8 = 2 \times 4 + 0 \\ 2 = 0 \times 4 + 2 \end{array}$$

Solution

$$(1022)_3 = (203)_4$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Base 3 to Base 4 (2)

Question

$$(1022)_3 = (?)_4$$

Solution 2 (Fluency with radix-3 is assumed)

$$(1022)_3 = (22)_3 \times (11)_3 + (10)_3$$

$$(22)_3 = 2 \times (11)_3 + 0$$

$$2 = 0 \times (11)_3 + 2$$

Solution

$$(1022)_3 = (203)_4$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Base 12 to Base 16 (1)

Question

$$(A5)_{12} = (?)_{16}$$

Solution 1 (Fluency with radix-10 is assumed)

We split the work into two steps:

$$(A5)_{12} = x \text{ then } x = (?)_{16}$$

Convert to decimal

$$\begin{aligned}x &= A \times 12 + \\ &+ 5 \times 1 = \\ &= 125\end{aligned}$$

Convert to base 16

$$\begin{aligned}125 &= 7 \times 16 + 13 \\ 7 &= 0 \times 16 + 7\end{aligned}$$

Solution

$$(A5)_{12} = (7D)_{16}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Base 12 to Base 16 (2)

Question

$$(A5)_{12} = (?)_{16}$$

Solution 2 (Fluency with radix-12 is assumed)

$$(A5)_{12} = 7_{12} \times (14)_{12} + (11)_{12}$$

$$(7)_{12} = 0 \times (14)_{12} + 7$$

Solution

$$(A5)_{16} = (7D)_{16}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to Binary

Question

$$200 = (?)_2$$

Solution

$$200 = 100 \times 2 + 0$$

$$100 = 50 \times 2 + 0$$

$$50 = 25 \times 2 + 0$$

$$25 = 12 \times 2 + 1$$

$$12 = 6 \times 2 + 0$$

$$6 = 3 \times 2 + 0$$

$$3 = 1 \times 2 + 1$$

$$1 = 0 \times 2 + 1$$

Solution

$$100 = (11001000)_2$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Binary to Decimal (1)

Question

$$(111001)_2 = ?$$

Solution 1 (Fluency in decimal is assumed)

$$1 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = \\ 32 + 16 + 8 + 1 = 57$$

Solution

$$(111001)_2 = 57$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Binary to Decimal (2)

Question

$$(111001)_2 = ?$$

Solution 2 (Fluency in binary is assumed)

$$(111001)_2 = (101)_2 \times (1010)_2 + (111)_2$$

$$(101)_2 = 0 \times (1010)_2 + (101)_2$$

Solution

$$(111001)_2 = 57$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Switching Radices, Special Case

- It is rather clear that switching radix requires work
- When working in binary, we have **long** numbers
- Converting these numbers to decimal and back is not friendly

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

The Easy Switching Theorem

Lemma

Assume $c = b^k$, $b, k \in \mathbb{N}$, $k \geq 1$ and $b > 1$. Let

$$x = (d_{n-1} \cdots d_0)_c.$$

Then

$$x = (e_{m-1} \cdots e_0)_b,$$

where

$$m = n \times k$$

and for each $i < n$,

$$d_i = (e_{i \times k + k - 1} \cdots e_{i \times k + 1} e_{i \times k})_b.$$

$\mathbb{N}$

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

$\mathbb{Z}$

Complement

Hardware

$\mathbb{Q}$

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Proof of the Easy Switching Theorem

$$\begin{aligned}x &= \sum_{i=0}^{n-1} d_i \times c^i = \sum_{i=0}^{n-1} d_i \times b^{i \times k} = \\&= \sum_{i=0}^{n-1} \left(\sum_{j=0}^{k-1} e_{i \times k + j} \times b^j \right) \times b^{i \times k} = \\&= \sum_{i=0}^{n-1} \left(\sum_{j=0}^{k-1} e_{i \times k + j} \times b^{i \times k + j} \right) = \sum_{i=0}^{m-1} e_i \times b^i.\end{aligned}$$



N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Binary $\iff$ Hexa

$2^4 = 16$, hence four binary digits are one hex digit

Binary	0100101111000001
Hexa	

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Binary $\iff$ Hexa

$2^4 = 16$, hence four binary digits are one hex digit

Binary	0100101111000001
Hexa	

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Binary $\iff$ Hexa

$2^4 = 16$, hence four binary digits are one hex digit

Binary	01001011		1100		0001
Hexa					

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Binary $\iff$ Hexa

$2^4 = 16$, hence four binary digits are one hex digit

Binary	0100	1011	1100	0001
Hexa				

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Binary $\iff$ Hexa

$2^4 = 16$, hence four binary digits are one hex digit

Binary	0100	1011	1100	0001
Hexa	4	B	C	1

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Binary $\iff$ Hexa

$2^4 = 16$, hence four binary digits are one hex digit

Binary	0100	1011	1100	0001
Hexa	4	B	C	1

Binary	10100110100111
Hexa	

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Binary $\iff$ Hexa

$2^4 = 16$, hence four binary digits are one hex digit

Binary	0100	1011	1100	0001
Hexa	4	B	C	1

Binary	1010011010	0111
Hexa		

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Binary $\iff$ Hexa

$2^4 = 16$, hence four binary digits are one hex digit

Binary	0100	1011	1100	0001
Hexa	4	B	C	1

Binary	101001	1010	0111
Hexa			

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Binary $\iff$ Hexa

$2^4 = 16$, hence four binary digits are one hex digit

Binary	0100	1011	1100	0001
Hexa	4	B	C	1

Binary	10	1001	1010	0111
Hexa				

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Binary $\iff$ Hexa

$2^4 = 16$, hence four binary digits are one hex digit

Binary	0100	1011	1100	0001
Hexa	4	B	C	1

Binary	10	1001	1010	0111
Hexa	2	9	A	7

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Radix-3 $\iff$ Radix-9

$3^2 = 9$, hence two radix-3 digits are one radix-9 digit

Radix-3 12021022

Radix-9

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Radix-3 $\iff$ Radix-9

$3^2 = 9$, hence two radix-3 digits are one radix-9 digit

Radix-3	12021022
Radix-9	

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Radix-3 $\iff$ Radix-9

$3^2 = 9$, hence two radix-3 digits are one radix-9 digit

Radix-3	1202	10	22
Radix-9			

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Radix-3 $\iff$ Radix-9

$3^2 = 9$, hence two radix-3 digits are one radix-9 digit

Radix-3	12	02	10	22
Radix-9				

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Radix-3 $\iff$ Radix-9

$3^2 = 9$, hence two radix-3 digits are one radix-9 digit

Radix-3	1	2	0	2	1	0	2	2
Radix-9	5	2	3	8				

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Radix-3 $\iff$ Radix-9

$3^2 = 9$, hence two radix-3 digits are one radix-9 digit

Radix-3	1	2	0	2	1	0	2	2
Radix-9	5	2	3	8				

Radix-3	2	0	2	1	0	2	2
Radix-9							

N

- Peano
- Caveperson
- Addition
- Subtraction
- Multiplication
- Switching radices
- Hardware

Z

- Complement
- Hardware

Q

- Examples
- Hardware
- Fixed point
- BCD
- Floating-point
- Examples

Example: Radix-3 $\iff$ Radix-9

$3^2 = 9$, hence two radix-3 digits are one radix-9 digit

Radix-3	12 02 10 22
Radix-9	5 2 3 8

Radix-3	20210 22
Radix-9	

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Radix-3 $\iff$ Radix-9

$3^2 = 9$, hence two radix-3 digits are one radix-9 digit

Radix-3	12 02 10 22
Radix-9	5 2 3 8

Radix-3	202 10 22
Radix-9	

N

- Peano
- Caveperson
- Addition
- Subtraction
- Multiplication
- Switching radices
- Hardware

Z

- Complement
- Hardware

Q

- Examples
- Hardware
- Fixed point
- BCD
- Floating-point
- Examples

Example: Radix-3 $\iff$ Radix-9

$3^2 = 9$, hence two radix-3 digits are one radix-9 digit

Radix-3	12	02	10	22
Radix-9	5	2	3	8

Radix-3	20	21	02	22
Radix-9				

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Radix-3 $\iff$ Radix-9

$3^2 = 9$, hence two radix-3 digits are one radix-9 digit

Radix-3	1	2	0	2	1	0	2	2
Radix-9	5	2	3	8				

Radix-3	2	0	2	1	0	2	2	
Radix-9	2	2	3	8				

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Recall

- Number does not have a base
- Number notation has a base

$\mathbb{N}$ and the Hardware

$\mathbb{N}$

- Peano
- Caveperson
- Addition
- Subtraction
- Multiplication
- Switching radices
- Hardware

$\mathbb{Z}$

- Complement
- Hardware

$\mathbb{Q}$

- Examples
- Hardware
- Fixed point
- BCD
- Floating-point
- Examples

- The closest to $\mathbb{N}$ is unsigned char/short/long
- The '+' and '×' are exact if there is no over/underflow
- E.g.,

```
unsigned char c = 255;  
c = c + 1;
```

- c will be 0 at the end of the above snippet
- The number of (binary) digits is **fixed**

type	binary digits
unsigned char	8
unsigned short	16
unsigned long	32
unsigned long long	64

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

For numbers with n bits

- There is overflow if $c_n = 1$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Zero Extension

```
unsigned char  c = 5; // c=0x05  
unsigned short s = c; // s=0x005
```

```
unsigned char  c = 251; // c=0xFB  
unsigned short s = c;   // s=0x00FB
```

Contrast this slide with the corresponding slide for $\mathbb{Z}$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

$\mathbb{Z}$

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

- Variables are, in some sense, abstract as the naturals
- We do not **know** their notation
- We **believe** the manufacturer
- We have no access to the internals

So, how can we get the internal notation?

- We cannot
- We can get **digits** in some arbitrary radix
- We **believe** using radix 2 will give the internal structure

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Digits (right to left)

```
void digits(unsigned long x,  
            unsigned long b) {  
    assert (b>1);  
    do {  
        unsigned long d = x % b;  
        x = x / b;  
        printf("%d", d); // Digits (decimal)  
    } while (x > 0);  
}
```

The division with remainder theorem in action

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

The Integers ($\mathbb{Z}$)

Whoever is happy with the integers can jump to [Notation](#)

N

- Peano
- Caveperson
- Addition
- Subtraction
- Multiplication
- Switching radices
- Hardware

Z

- Complement
- Hardware

Q

- Examples
- Hardware
- Fixed point
- BCD
- Floating-point
- Examples

Definition

The relation $\sim$ over $\mathbb{N} \times \mathbb{N}$ is defined as follows:

$$\langle m_0, n_0 \rangle \sim \langle m_1, n_1 \rangle \iff m_0 + n_1 = n_0 + m_1$$

Claim

$\sim$ is an equivalence relation

Definition

$$\mathbb{Z} = \mathbb{N} \times \mathbb{N} / \sim$$

Definition

$$0 = [\langle 0, 0 \rangle]$$

Definition (Embedding $\mathbb{N} \rightarrow \mathbb{Z}$)

$$n \mapsto [\langle n, 0 \rangle]$$

 $\mathbb{N}$

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

 $\mathbb{Z}$

Complement
Hardware

 $\mathbb{Q}$

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Definition

- $\langle m_0, n_0 \rangle + \langle m_1, n_1 \rangle = \langle m_0 + m_1, n_0 + n_1 \rangle$
- $\langle m_0, n_0 \rangle \times \langle m_1, n_1 \rangle =$
 $\langle m_0 \times m_1 + n_0 \times n_1, m_0 \times n_1 + n_0 \times m_1 \rangle$

Claim

'+' and '×' are compatible with the equivalence classes

Definition

- $[\langle m_0, n_0 \rangle] + [\langle m_1, n_1 \rangle] = [\langle m_0 + m_1, n_0 + n_1 \rangle]$
- $[\langle m_0, n_0 \rangle] \times [\langle m_1, n_1 \rangle] =$
 $[\langle m_0 \times m_1 + n_0 \times n_1, m_0 \times n_1 + n_0 \times m_1 \rangle]$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Note

$$[\langle m, n \rangle] + [\langle n, m \rangle] = [\langle 0, 0 \rangle] = 0$$

Definition

$$-[\langle m, n \rangle] = [\langle n, m \rangle]$$

We just got the negatives!

- We identify $\langle [n, 0] \mid n \in \mathbb{N} \rangle$ with $\mathbb{N}$
- The proper negatives are $\langle [0, n] \mid n \in \mathbb{N} \setminus \{0\} \rangle$

 $\mathbb{N}$

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

 $\mathbb{Z}$

Complement
Hardware

 $\mathbb{Q}$

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Signed-Magnitude Notation

- $x \geq 0$: Use the $\mathbb{N}$ -notation with an optional '+' prefixed
- $x < 0$: Use the $\mathbb{N}$ -notation with '-' prefixed

Operations can be translated to $\mathbb{N}$

$a + b$	$b \geq 0$	$b < 0$
$a \geq 0$	$a + b$	$a - (-b)$
$a < 0$	$b - (-a)$	$-((-a) + (-b))$

$a \times b$	$b \geq 0$	$b < 0$
$a \geq 0$	$a \times b$	$-(a \times (-b))$
$a < 0$	$-((-a) \times b)$	$(-a) \times (-b)$

when $b > a > 0$
 $a - b = -(b - a)$

We can keep the operations from the $\mathbb{N}$ -notation

$\mathbb{N}$

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

$\mathbb{Z}$

Complement
Hardware

$\mathbb{Q}$

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Binary Signed-Magnitude

Not used by hardware for integers

- Left most bit is the sign
 - ▶ 1 for negative
 - ▶ 0 for non-negative
- We have $+0$ and -0

Four bits example

0000	0	1000	-0
0001	1	1001	-1
0010	2	1010	-2
0011	3	1011	-3
0100	4	1100	-4
0101	5	1101	-5
0110	6	1110	-6
0111	7	1111	-7

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

2's Complement Motivation

Five bits

- $9 + (-9) = 0$
- $9 = (01001)_2$
-

$$\begin{array}{r} 01001 \\ +10111 \\ \hline 00000 \end{array}$$

- $-9 = (10111)_2$
- The number of bits **must** be **FIXED**

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Calculating the 2's Complement

Five bits

- Represent -11
- $11 = 01011$
- Method 1

$$\begin{array}{r} 100000 \\ - 01011 \\ \hline 10101 \end{array}$$

- Method 2

$$\begin{array}{r} 11111 \\ - 01011 \\ \hline 10100 \\ + 1 \\ \hline 10101 \end{array}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

2's Complement to Decimal

Five bits

- 00110
 - ▶ Left most bit is zero. Usual binary: 6
- 11001
 - ▶ Left most bit is one. Calculate 2's complement:

$$\begin{array}{r} 100000 \\ - 11001 \\ \hline 00110 \end{array}$$

Hence: -6

- Note the pathology at 10000

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Four bits example

0000	0	1000	-8
0001	1	1001	-7
0010	2	1010	-6
0011	3	1011	-5
0100	4	1100	-4
0101	5	1101	-3
0110	6	1110	-2
0111	7	1111	-1

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

- This method works when the number of digits is **fixed**
- The method is based on the overflow in the unsigned case:

▶ 4-digits: $(1111)_2 + 1_2 = (0000)_2$

- The weight of the leftmost digit is negative
- The meaning of $(d_{n-1} \cdots d_0)_b$ is

$$-d_{n-1} \times b^{n-1} + \sum_{i=0}^{n-2} d_i \times b^i$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Addition examples

	unsigned	signed		unsigned	signed
0011	3	3	0011	3	3
+1000	+8	+ - 8	+1111	+15	+ - 1
1011	11	-5	0010	2	2
	✓	✓		X	✓
0111	7	7	1000	8	-8
+0001	+1	+1	+1111	+15	-1
1000	8	-8	0111	7	7
	✓	X		X	X

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

(Carmi) Lecture 2 reached here

N

- Peano
- Caveperson
- Addition
- Subtraction
- Multiplication
- Switching radices
- Hardware

Z

- Complement
- Hardware

Q

- Examples
- Hardware
- Fixed point
- BCD
- Floating-point
- Examples

$\mathbb{Z}$ and the Hardware

N

- Peano
- Caveperson
- Addition
- Subtraction
- Multiplication
- Switching radices
- Hardware

Z

- Complement
- Hardware**

Q

- Examples
- Hardware
- Fixed point
- BCD
- Floating-point
- Examples

What was said about $\mathbb{N}$ hold also for $\mathbb{Z}$

The hardware stores integers in 2's-complement notation

- **Not** in signed-magnitude notation
- We need to trust the manufacturers on this
- There are hints of this, of course

It seems the reason for the 2s-complement is historic

- Addition/subtraction algorithms are identical for unsigned and signed
- Thus saving logic and machine instructions

$\mathbb{N}$

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

$\mathbb{Z}$

Complement
Hardware

$\mathbb{Q}$

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

- The closest to $\mathbb{Z}$ is char/short/long
- The '+' and '×' are exact if there is no over/underflow
- E.g.,

```
char c = 127;  
c = c + 1;
```

- c will be -128 at the end of the above snippet
- The number of (binary) digits is **fixed**

type	binary digits
char	8
short	16
long	32
long long	64

 $\mathbb{N}$

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

 $\mathbb{Z}$

Complement
Hardware

 $\mathbb{Q}$

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Assume n is the number of binary digits and c_i are the carry bits

- Unsigned overflow: $c_n = 1$
- Sign overflow: $c_n \neq c_{n-1}$

- All hardware we know of ignore overflow
- There is hardware exposing the information to the interested (e.g., x86)
- (No one ever look there)
- There is hardware which do not (e.g., riscv)

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Sign Extension

```
char    c = -5;    // c = 0xFB
short   s = c;      // s = 0xFFFB
```

```
char    c = 5;     // c = 0x05
short   s = c;      // s = 0x0005
```

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Digits (right to left)

```
void sdigits(long x,  
             unsigned long b) {  
    assert (b>1);  
    if (x >= 0)  
        digits((unsigned long)x, b);  
    else {  
        digits((unsigned long)(-x), b);  
        printf("-");  
    }  
}
```

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

The Rationals ($\mathbb{Q}$)

There is no direct type in conventional programming languages for $\mathbb{Q}$

Whoever is happy with the rationals can jump to [Notation](#)

$\mathbb{N}$

- Peano
- Caveperson
- Addition
- Subtraction
- Multiplication
- Switching radices
- Hardware

$\mathbb{Z}$

- Complement
- Hardware

$\mathbb{Q}$

- Examples
- Hardware
- Fixed point
- BCD
- Floating-point
- Examples

Of course, the following is formalizing the obvious

Definition ($\sim$)

The relation $\sim$ over $\mathbb{Z} \times (\mathbb{Z} \setminus \{0\})$ is defined by

$$\langle m_0, n_0 \rangle \sim \langle m_1, n_1 \rangle \iff m_0 \times n_1 = n_0 \times m_1$$

Definition

$$\mathbb{Q} = (\mathbb{Z} \times (\mathbb{Z} \setminus \{0\})) / \sim$$

Definition

$$\frac{m}{n} = [\langle m, n \rangle]$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Definition

$$\frac{m_0}{n_0} + \frac{m_1}{n_1} = \left[\frac{m_0 \times n_1 + m_1 \times n_0}{n_0 \times n_1} \right]$$

Definition

$$\frac{m_0}{n_0} \times \frac{m_1}{n_1} = \left[\frac{m_0 \times m_1}{n_0 \times n_1} \right]$$

Definition (Embedding $\mathbb{Z} \rightarrow \mathbb{Q}$)

$$n \mapsto \frac{n}{1}$$

Lots of things to prove above!

 $\mathbb{N}$

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

 $\mathbb{Z}$

Complement
Hardware

 $\mathbb{Q}$

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Lemma

Assume $0 \leq x < 1$, $b \in \mathbb{N}$ and $b > 1$. Then there is unique $d \in \mathbb{N}$ and $f < 1$, where $d < b$, such that

$$x \times b = d + f.$$

Corollary

Assume $0 \leq x < 1$, $b \in \mathbb{N}$ and $b > 1$. Then there is a sequence $\{d_n\}_{n=-1}^{-\infty}$ such that

$$x = \sum_{n=-1}^{-\infty} d_n \times b^n.$$

$0.\{d_n\}_{n=-1}^{-\infty}$ is the b -radix notation of x

 $\mathbb{N}$

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

 $\mathbb{Z}$

Complement
Hardware

 $\mathbb{Q}$

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Definition

A notation $0.(d_n)_{n=-1}^{-\infty}$ is said to be repetitive if there is $k, l \in \mathbb{N}$ such that for each $i \in \mathbb{N}$ and $j < l$, $d_{k+i \times l+j} = d_{k+j}$

Lemma

The b -radix notation of $x \in \mathbb{Q}$, $0 \leq x < 1$ is repetitive.

Proof.

For each d_n there is a corresponding f_n . For each f_n there is a corresponding r_n such that $f_n = \frac{r_n}{m}$. Thus after at most m steps there is k, n such that $r_k = r_n$, hence $f_k = f_n$ and we find the repetition. $\square$

Notation

$$0.d_1 \cdots d_k \overline{d_{k+1} \cdots d_n} = 0.d_1 \cdots d_k d_{k+1} \cdots d_n d_{k+1} \cdots d_n \cdots$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Fraction to Decimal

Question

$$\frac{7}{25} = (?)_{10}$$

Solution

$$\begin{aligned}\frac{7}{25} \times 10 &= 2 + \frac{20}{25} \\ \frac{20}{25} \times 10 &= 8 + 0\end{aligned}$$

Thus $\frac{7}{25} = 0.28$.

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples

Hardware
Fixed point
BCD
Floating-point
Examples

Example: Radix-5 to Decimal (1)

Question

$$(0.34)_5 = (?)_{10}$$

Solution 1

$$(0.34)_5 = \frac{3}{5} + \frac{4}{25} = \frac{19}{25}$$

then

$$\frac{19}{25} \times 10 = 7 + \frac{15}{25}$$

$$\frac{15}{25} \times 10 = 6 + 0$$

Thus $(0.34)_5 = 0.76$.

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples

Hardware
Fixed point
BCD
Floating-point
Examples

Example: Radix-5 fraction to Decimal (2)

Question

$$(0.34)_5 = (?)_{10}$$

Solution 2

$$(0.34)_5 \times (20)_5 = (12)_5 + (0.3)_5$$

$$(0.3)_5 \times (20)_5 = (11)_5 + 0$$

$$\text{Thus } (0.34)_5 = 0.76$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples

Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to Radix-5

Question

$$0.5 = (?)_5$$

Solution

$$0.5 \times 5 = 2 + 0.5 \quad // \text{ We are in infinite loop}$$

Thus $0.5 = (0.222\ldots)_5 = (0.\overline{2})_5$.

Imprecision is almost inevitable

since we do not possess infinite memories

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples

Hardware
Fixed point
BCD
Floating-point
Examples

Example: Fraction to Binary

Question

$$\frac{1}{5} = (?)_2$$

Solution

$$\frac{1}{5} \times 2 = 0 + \frac{2}{5}$$

$$\frac{2}{5} \times 2 = 0 + \frac{4}{5}$$

$$\frac{4}{5} \times 2 = 1 + \frac{3}{5}$$

$$\frac{3}{5} \times 2 = 1 + \frac{1}{5} \text{ here comes the infinite loop}$$

Thus $\frac{1}{5} = (0.\overline{0011})_2$.

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples

Hardware
Fixed point
BCD
Floating-point
Examples

Example: Fraction to Binary

Question

$$\frac{19}{28} = (?)_2$$

Solution

$$\begin{array}{lcl} \frac{19}{28} \times 2 = 1 + \frac{10}{28} & \frac{20}{28} \times 2 = 1 + \frac{12}{28} & \\ \frac{10}{28} \times 2 = 0 + \frac{20}{28} & \frac{12}{28} \times 2 = 0 + \frac{24}{28} & \frac{19}{28} = (0.10\overline{101})_2 \\ & \frac{24}{28} \times 2 = 1 + \frac{20}{28} & \end{array}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples

Hardware
Fixed point
BCD
Floating-point
Examples

$\mathbb{Q}$ and the Hardware

$\mathbb{N}$

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

$\mathbb{Z}$

Complement

Hardware

$\mathbb{Q}$

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

- The operations can be reduced to $\mathbb{Z}$ -operations
- Thus $\mathbb{Z}$ notation claims can be used.

$$(0.3)_4 + (1.25)_4 = \frac{(30)_4 + (125)_4}{(100)_4}$$
$$(0.3)_4 \times (1.25)_4 = \frac{(3)_4 \times (125)_4}{(1000)_4}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

- Computer memory is finite
- Existence of maximum and minimum, as in the integers, is obvious
- In addition $\mathbb{Q}$ might require infinite digits
- Thus in addition to overflow and underflow we have imprecise calculations

$\mathbb{N}$

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

$\mathbb{Z}$

Complement
Hardware

$\mathbb{Q}$

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

- Fixed point
 - ▶ Never really processor implemented
 - ▶ Non-processor circuits might implement it
- Binary Coded Decimal
 - ▶ Ancient hardware had it
 - ▶ Software only on modern hardware
 - ▶ Important
- Floating point
 - ▶ Hardware implemented
 - ▶ Important

$\mathbb{N}$

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

$\mathbb{Z}$

Complement
Hardware

$\mathbb{Q}$

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Fixed Point

Left for the future

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Binary Coded Decimal

Left for the future

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

(Binary) Floating-Point Notation

Decimal FP is left for the future

These are all rationals, actually

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

- Sometime we need some precision in a vast range
- Numbers are represented in the form

$$m \times 10^n,$$

where $|m| < 10$.

Examples

3.14159

6.02×10^{23}

8.407564×10^{-16}

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

- First standard 1985
- The driving force was Intel
- Last standard as of yet, 2019
- Before 1985 manufacturer used propriety formats
- (Also after, e.g., IBM added IEEE 754 circa 2000)
- (float calculations were manufacturer dependent!)

N

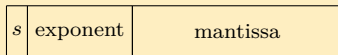
Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples



The exponent is considered to be an unsigned number of w -bits width

Let $\text{bias} = 2^{w-1} - 1$

- **Normal form** $0 < \text{exponent} < 2^w - 1$:

$$(-1)^s \times 1.\text{mantissa} \times 2^{\text{exponent} - \text{bias}}$$

- **Subnormal form** $\text{exponent} = 0$:

$$(-1)^s \times 0.\text{mantissa} \times 2^{1 - \text{bias}}$$

- **Bizzaro** $\text{exponent} = 2^w - 1$: NaNs, infinities, et. al.

We ignore this case in this course

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Binary FP fields width

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

	width of		
	<i>s</i>	exponent	mantissa
binary16	1	5	10
binary32	1	8	23
binary64	1	11	52
binary128	1	15	112
binary256	1	18	237

float

double

Example: Decimal to binary16

Question

Present 200 in binary16

|

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary16

Question

Present 200 in binary16

200 | 0

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary16

Question

Present 200 in binary16

200		0
100		0

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary16

Question

Present 200 in binary16

200		0
100		0
50		0

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary16

Question

Present 200 in binary16

200	0
100	0
50	0
25	1

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary16

Question

Present 200 in binary16

200	0
100	0
50	0
25	1
12	0

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary16

Question

Present 200 in binary16

200	0
100	0
50	0
25	1
12	0
6	0

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary16

Question

Present 200 in binary16

200	0
100	0
50	0
25	1
12	0
6	0
3	1

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary16

Question

Present 200 in binary16

200	0
100	0
50	0
25	1
12	0
6	0
3	1
1	1

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary16

Question

Present 200 in binary16

200	0
100	0
50	0
25	1
12	0
6	0
3	1
1	1
0	

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary16

Question

Present 200 in binary16

200		0
100		0
50		0
25		1
12		0
6		0
3		1
1		1
0		

$$200 = (11001000)_2$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary16

Question

Present 200 in binary16

200	0
100	0
50	0
25	1
12	0
6	0
3	1
1	1
0	

$$200 = (11001000)_2$$

$$(11001000)_2 = (1.1001000)_2 \times 2^7$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary16

Question

Present 200 in binary16

200	0
100	0
50	0
25	1
12	0
6	0
3	1
1	1
0	

$$200 = (11001000)_2$$

$$(11001000)_2 = (1.1001000)_2 \times 2^7$$

$$s = 0$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary16

Question

Present 200 in binary16

200	0
100	0
50	0
25	1
12	0
6	0
3	1
1	1
0	

$$200 = (11001000)_2$$

$$(11001000)_2 = (1.1001000)_2 \times 2^7$$

$$s = 0$$

$$\text{exp} = 7 + 15 = 22 = (10110)_2$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary16

Question

Present 200 in binary16

200	0
100	0
50	0
25	1
12	0
6	0
3	1
1	1
0	

$$200 = (11001000)_2$$

$$(11001000)_2 = (1.1001000)_2 \times 2^7$$

$$s = 0$$

$$\text{exp} = 7 + 15 = 22 = (10110)_2$$

$$\text{mantissa} = (1001000000)_2$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary16

Question

Present 200 in binary16

200	0	$200 = (11001000)_2$
100	0	$(11001000)_2 = (1.1001000)_2 \times 2^7$
50	0	
25	1	$s = 0$
12	0	$\text{exp} = 7 + 15 = 22 = (10110)_2$
6	0	$\text{mantissa} = (1001000000)_2$
3	1	
1	1	
0		

15 14

10 9

0

0	1	0	1	1	0	1	0	0	1	0	0	0	0	0	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary16

Question

Present 200 in binary16

200		0	$200 = (11001000)_2$
100		0	$(11001000)_2 = (1.1001000)_2 \times 2^7$
50		0	$s = 0$
25		1	$\text{exp} = 7 + 15 = 22 = (10110)_2$
12		0	$\text{mantissa} = (1001000000)_2$
6		0	
3		1	
1		1	
0			

15 14

10 9

0

0	1	0	1	1	0	1	0	0	1	0	0	0	0	0	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

$$200 = (0101101001000000)_2 \text{bin16} = (5A40)_{16} \text{bin16}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary16

Question

Present $\frac{1}{5}$ in binary16

|

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary16

Question

Present $\frac{1}{5}$ in binary16

$$\frac{1}{5} \mid 0$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary16

Question

Present $\frac{1}{5}$ in binary16

$$\begin{array}{r|l} \frac{1}{5} & 0 \\ \frac{2}{5} & 0 \end{array}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary16

Question

Present $\frac{1}{5}$ in binary16

$$\begin{array}{r|l} \frac{1}{5} & 0 \\ \frac{2}{5} & 0 \\ \frac{4}{5} & 1 \end{array}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary16

Question

Present $\frac{1}{5}$ in binary16

$$\begin{array}{r|l} \frac{1}{5} & 0 \\ \frac{2}{5} & 0 \\ \frac{4}{5} & 1 \\ \frac{3}{5} & 1 \end{array}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary16

Question

Present $\frac{1}{5}$ in binary16

$$\begin{array}{r|l} \frac{1}{5} & 0 \\ \frac{2}{5} & 0 \\ \frac{4}{5} & 1 \\ \frac{3}{5} & 1 \\ \frac{1}{5} & \end{array}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary16

Question

Present $\frac{1}{5}$ in binary16

$$\begin{array}{r|l} \frac{1}{5} & 0 \\ \frac{2}{5} & 0 \\ \frac{4}{5} & 1 \\ \frac{3}{5} & 1 \\ \frac{1}{5} & \end{array}$$

$$\frac{1}{5} = (0.\overline{0011})_2 = (1.\overline{10011})_2 \times 2^{-3}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary16

Question

Present $\frac{1}{5}$ in binary16

$$\begin{array}{r|l} \frac{1}{5} & 0 \\ \frac{2}{5} & 0 \\ \frac{4}{5} & 1 \\ \frac{3}{5} & 1 \\ \frac{1}{5} & \end{array}$$

$$\frac{1}{5} = (0.\overline{0011})_2 = (1.\overline{10011})_2 \times 2^{-3}$$
$$s = 0$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary16

Question

Present $\frac{1}{5}$ in binary16

$\frac{1}{5}$	0
$\frac{2}{5}$	0
$\frac{4}{5}$	1
$\frac{3}{5}$	1
$\frac{1}{5}$	

$$\frac{1}{5} = (0.\overline{0011})_2 = (1.\overline{10011})_2 \times 2^{-3}$$

$$s = 0$$

$$\text{exponent} = -3 + 15 = 12 = (01100)_2$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary16

Question

Present $\frac{1}{5}$ in binary16

$\frac{1}{5}$	0
$\frac{2}{5}$	0
$\frac{4}{5}$	1
$\frac{3}{5}$	1
$\frac{1}{5}$	

$$\frac{1}{5} = (0.\overline{0011})_2 = (1.\overline{10011})_2 \times 2^{-3}$$

$$s = 0$$

$$\text{exponent} = -3 + 15 = 12 = (01100)_2$$

$$\text{mantissa} = (1001100110)_2$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary16

Question

Present $\frac{1}{5}$ in binary16

$\frac{1}{5}$		0
$\frac{2}{5}$		0
$\frac{4}{5}$		1
$\frac{3}{5}$		1
$\frac{1}{5}$		

$$\frac{1}{5} = (0.\overline{0011})_2 = (1.\overline{10011})_2 \times 2^{-3}$$

$$s = 0$$

$$\text{exponent} = -3 + 15 = 12 = (01100)_2$$

$$\text{mantissa} = (1001100110)_2$$

15 14		10 9		0								
0	0	1	1	0	0	1	1	0	0	1	1	0

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary16

Question

Present $\frac{1}{5}$ in binary16

$\frac{1}{5}$	0
$\frac{2}{5}$	0
$\frac{4}{5}$	1
$\frac{3}{5}$	1
$\frac{1}{5}$	

$$\frac{1}{5} = (0.\overline{0011})_2 = (1.\overline{10011})_2 \times 2^{-3}$$

$$s = 0$$

$$\text{exponent} = -3 + 15 = 12 = (01100)_2$$

$$\text{mantissa} = (1001100110)_2$$

15 14		10 9						0							
0	0	1	1	0	0	1	0	0	1	1	0	0	1	1	0

$$\frac{1}{5} \approx (0011001001100110)_2 \text{bin16} = (3266)_{16} \text{bin16}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

(Carmi) Lecture 3 reached here

N

- Peano
- Caveperson
- Addition
- Subtraction
- Multiplication
- Switching radices
- Hardware

Z

- Complement
- Hardware

Q

- Examples
- Hardware
- Fixed point
- BCD
- Floating-point
- Examples

Example: Decimal to binary₁₆

Present 0.00006103515625 in binary₁₆



N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary16

Present 0.00006103515625 in binary16

0.00006103515625 | 0 |

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary16

Present 0.00006103515625 in binary16

0.00006103515625		0	
0.0001220703125		0	

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary₁₆

Present 0.00006103515625 in binary₁₆

0.00006103515625	0	
0.0001220703125	0	
0.000244140625	0	

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary16

Present 0.00006103515625 in binary16

0.00006103515625	0	
0.0001220703125	0	
0.000244140625	0	
0.00048828125	0	

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary16

Present 0.00006103515625 in binary16

0.00006103515625	0	
0.0001220703125	0	
0.000244140625	0	
0.00048828125	0	
0.0009765625	0	

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary16

Present 0.00006103515625 in binary16

0.00006103515625	0	
0.0001220703125	0	
0.000244140625	0	
0.00048828125	0	
0.0009765625	0	
0.001953125	0	

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary₁₆

Present 0.00006103515625 in binary₁₆

0.00006103515625		0
0.0001220703125		0
0.000244140625		0
0.00048828125		0
0.0009765625		0
0.001953125		0
0.00390625		0

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Example: Decimal to binary16

Present 0.00006103515625 in binary16

0.00006103515625	0
------------------	---

0.0001220703125	0
-----------------	---

0.000244140625	0
----------------	---

0.00048828125	0
---------------	---

0.0009765625	0
--------------	---

0.001953125	0
-------------	---

0.00390625	0
------------	---

0.0078125	0
-----------	---

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary₁₆

Present 0.00006103515625 in binary₁₆

0.00006103515625	0
0.0001220703125	0
0.000244140625	0
0.00048828125	0
0.0009765625	0
0.001953125	0
0.00390625	0
0.0078125	0
0.015625	0

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary₁₆

Present 0.00006103515625 in binary₁₆

0.00006103515625	0	
------------------	---	--

0.0001220703125	0	
-----------------	---	--

0.000244140625	0	
----------------	---	--

0.00048828125	0	
---------------	---	--

0.0009765625	0	
--------------	---	--

0.001953125	0	
-------------	---	--

0.00390625	0	
------------	---	--

0.0078125	0	
-----------	---	--

0.015625	0	
----------	---	--

0.03125	0	
---------	---	--

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary₁₆

Present 0.00006103515625 in binary₁₆

0.00006103515625	0	
0.0001220703125	0	
0.000244140625	0	
0.00048828125	0	
0.0009765625	0	
0.001953125	0	
0.00390625	0	
0.0078125	0	
0.015625	0	
0.03125	0	
0.0625	0	

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary₁₆

Present 0.00006103515625 in binary₁₆

0.00006103515625	0
0.0001220703125	0
0.000244140625	0
0.00048828125	0
0.0009765625	0
0.001953125	0
0.00390625	0
0.0078125	0
0.015625	0
0.03125	0
0.0625	0
0.125	0

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary₁₆

Present 0.00006103515625 in binary₁₆

0.00006103515625	0
0.0001220703125	0
0.000244140625	0
0.00048828125	0
0.0009765625	0
0.001953125	0
0.00390625	0
0.0078125	0
0.015625	0
0.03125	0
0.0625	0
0.125	0
0.25	0

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary₁₆

Present 0.00006103515625 in binary₁₆

0.00006103515625	0
0.0001220703125	0
0.000244140625	0
0.00048828125	0
0.0009765625	0
0.001953125	0
0.00390625	0
0.0078125	0
0.015625	0
0.03125	0
0.0625	0
0.125	0
0.25	0
0.5	1

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary16

Present 0.00006103515625 in binary16

0.00006103515625 | 0

0.0001220703125 | 0

0.000244140625 | 0

0.00048828125 | 0

0.0009765625 | 0

0.001953125 | 0

0.00390625 | 0

0.0078125 | 0

0.015625 | 0

0.03125 | 0

0.0625 | 0

0.125 | 0

0.25 | 0

0.5 | 1

$$0.00006103515625 = 1 \times 2^{-14}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Example: Decimal to binary16

Present 0.00006103515625 in binary16

0.00006103515625 | 0

0.0001220703125 | 0

0.000244140625 | 0

0.00048828125 | 0

0.0009765625 | 0

0.001953125 | 0

0.00390625 | 0

0.0078125 | 0

0.015625 | 0

0.03125 | 0

0.0625 | 0

0.125 | 0

0.25 | 0

0.5 | 1

$$0.00006103515625 = 1 \times 2^{-14}$$

$$s = 0$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Example: Decimal to binary16

Present 0.00006103515625 in binary16

0.00006103515625	0
0.0001220703125	0
0.000244140625	0
0.00048828125	0
0.0009765625	0
0.001953125	0
0.00390625	0
0.0078125	0
0.015625	0
0.03125	0
0.0625	0
0.125	0
0.25	0
0.5	1

$$0.00006103515625 = 1 \times 2^{-14}$$

$$s = 0$$

$$\text{exponent} = (00001)_2$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Example: Decimal to binary16

Present 0.00006103515625 in binary16

0.00006103515625	0
0.0001220703125	0
0.000244140625	0
0.00048828125	0
0.0009765625	0
0.001953125	0
0.00390625	0
0.0078125	0
0.015625	0
0.03125	0
0.0625	0
0.125	0
0.25	0
0.5	1

$$0.00006103515625 = 1 \times 2^{-14}$$

$$s = 0$$

$$\text{exponent} = (00001)_2$$

$$\text{mantissa} = (0000000000)_2$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary16

Present 0.00006103515625 in binary16

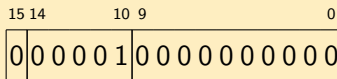
0.00006103515625	0
0.0001220703125	0
0.000244140625	0
0.00048828125	0
0.0009765625	0
0.001953125	0
0.00390625	0
0.0078125	0
0.015625	0
0.03125	0
0.0625	0
0.125	0
0.25	0
0.5	1

$$0.00006103515625 = 1 \times 2^{-14}$$

$$s = 0$$

$$\text{exponent} = (00001)_2$$

$$\text{mantissa} = (0000000000)_2$$



N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Example: Decimal to binary16

Present 0.00006103515625 in binary16

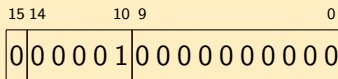
0.00006103515625	0
0.0001220703125	0
0.000244140625	0
0.00048828125	0
0.0009765625	0
0.001953125	0
0.00390625	0
0.0078125	0
0.015625	0
0.03125	0
0.0625	0
0.125	0
0.25	0
0.5	1

$$0.00006103515625 = 1 \times 2^{-14}$$

$$s = 0$$

$$\text{exponent} = (00001)_2$$

$$\text{mantissa} = (0000000000)_2$$



$$0.00006103515625 = (0400)_{16} \text{ bin16}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary₁₆

Present 0.000030517578125 in binary₁₆



N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary16

Present 0.000030517578125 in binary16

0.000030517578125 | 0 |

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary¹⁶

Present 0.000030517578125 in binary¹⁶

0.000030517578125		0	
0.00006103515625		0	

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary16

Present 0.000030517578125 in binary16

0.000030517578125	0	
0.00006103515625	0	
0.0001220703125	0	

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Example: Decimal to binary16

Present 0.000030517578125 in binary16

0.000030517578125	0
0.00006103515625	0
0.0001220703125	0
0.000244140625	0

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary16

Present 0.000030517578125 in binary16

0.000030517578125	0
0.00006103515625	0
0.0001220703125	0
0.000244140625	0
0.00048828125	0

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary16

Present 0.000030517578125 in binary16

0.000030517578125	0
0.00006103515625	0
0.0001220703125	0
0.000244140625	0
0.00048828125	0
0.0009765625	0

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary16

Present 0.000030517578125 in binary16

0.000030517578125	0
0.00006103515625	0
0.0001220703125	0
0.000244140625	0
0.00048828125	0
0.0009765625	0
0.001953125	0

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary16

Present 0.000030517578125 in binary16

0.000030517578125	0
0.00006103515625	0
0.0001220703125	0
0.000244140625	0
0.00048828125	0
0.0009765625	0
0.001953125	0
0.00390625	0

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Example: Decimal to binary16

Present 0.000030517578125 in binary16

0.000030517578125	0
0.00006103515625	0
0.0001220703125	0
0.000244140625	0
0.00048828125	0
0.0009765625	0
0.001953125	0
0.00390625	0
0.0078125	0

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary₁₆

Present 0.000030517578125 in binary₁₆

0.000030517578125	0
0.00006103515625	0
0.0001220703125	0
0.000244140625	0
0.00048828125	0
0.0009765625	0
0.001953125	0
0.00390625	0
0.0078125	0
0.015625	0

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Example: Decimal to binary16

Present 0.000030517578125 in binary16

0.000030517578125	0
0.00006103515625	0
0.0001220703125	0
0.000244140625	0
0.00048828125	0
0.0009765625	0
0.001953125	0
0.00390625	0
0.0078125	0
0.015625	0
0.03125	0

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Example: Decimal to binary₁₆

Present 0.000030517578125 in binary₁₆

0.000030517578125	0
0.00006103515625	0
0.0001220703125	0
0.000244140625	0
0.00048828125	0
0.0009765625	0
0.001953125	0
0.00390625	0
0.0078125	0
0.015625	0
0.03125	0
0.0625	0

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Example: Decimal to binary₁₆

Present 0.000030517578125 in binary₁₆

0.000030517578125	0
0.00006103515625	0
0.0001220703125	0
0.000244140625	0
0.00048828125	0
0.0009765625	0
0.001953125	0
0.00390625	0
0.0078125	0
0.015625	0
0.03125	0
0.0625	0
0.125	0

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary16

Present 0.000030517578125 in binary16

0.000030517578125	0
0.00006103515625	0
0.0001220703125	0
0.000244140625	0
0.00048828125	0
0.0009765625	0
0.001953125	0
0.00390625	0
0.0078125	0
0.015625	0
0.03125	0
0.0625	0
0.125	0
0.25	0

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Example: Decimal to binary16

Present 0.000030517578125 in binary16

0.000030517578125	0
0.00006103515625	0
0.0001220703125	0
0.000244140625	0
0.00048828125	0
0.0009765625	0
0.001953125	0
0.00390625	0
0.0078125	0
0.015625	0
0.03125	0
0.0625	0
0.125	0
0.25	0
0.5	1

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Example: Decimal to binary16

Present 0.000030517578125 in binary16

0.000030517578125	0	$0.000030517578125 = 1 \times 2^{-15}$
0.00006103515625	0	
0.0001220703125	0	
0.000244140625	0	
0.00048828125	0	
0.0009765625	0	
0.001953125	0	
0.00390625	0	
0.0078125	0	
0.015625	0	
0.03125	0	
0.0625	0	
0.125	0	
0.25	0	
0.5	1	

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: Decimal to binary16

Present 0.000030517578125 in binary16

0.000030517578125	0
0.00006103515625	0
0.0001220703125	0
0.000244140625	0
0.00048828125	0
0.0009765625	0
0.001953125	0
0.00390625	0
0.0078125	0
0.015625	0
0.03125	0
0.0625	0
0.125	0
0.25	0
0.5	1

$$0.000030517578125 = 1 \times 2^{-15}$$

$$0.000030517578125 = (0.1)_2 \times 2^{-14}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Example: Decimal to binary16

Present 0.000030517578125 in binary16

0.000030517578125	0
0.00006103515625	0
0.0001220703125	0
0.000244140625	0
0.00048828125	0
0.0009765625	0
0.001953125	0
0.00390625	0
0.0078125	0
0.015625	0
0.03125	0
0.0625	0
0.125	0
0.25	0
0.5	1

$$0.000030517578125 = 1 \times 2^{-15}$$

$$0.000030517578125 = (0.1)_2 \times 2^{-14}$$

$$s = 0$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Example: Decimal to binary16

Present 0.000030517578125 in binary16

0.000030517578125	0
0.00006103515625	0
0.0001220703125	0
0.000244140625	0
0.00048828125	0
0.0009765625	0
0.001953125	0
0.00390625	0
0.0078125	0
0.015625	0
0.03125	0
0.0625	0
0.125	0
0.25	0
0.5	1

$$0.000030517578125 = 1 \times 2^{-15}$$

$$0.000030517578125 = (0.1)_2 \times 2^{-14}$$

$$s = 0$$

$$\text{exponent} = (00000)_2$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Example: Decimal to binary16

Present 0.000030517578125 in binary16

0.000030517578125	0
0.00006103515625	0
0.0001220703125	0
0.000244140625	0
0.00048828125	0
0.0009765625	0
0.001953125	0
0.00390625	0
0.0078125	0
0.015625	0
0.03125	0
0.0625	0
0.125	0
0.25	0
0.5	1

$$0.000030517578125 = 1 \times 2^{-15}$$

$$0.000030517578125 = (0.1)_2 \times 2^{-14}$$

$$s = 0$$

$$\text{exponent} = (00000)_2$$

$$\text{mantissa} = (1000000000)_2$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Example: Decimal to binary16

Present 0.000030517578125 in binary16

0.000030517578125	0
0.00006103515625	0
0.0001220703125	0
0.000244140625	0
0.00048828125	0
0.0009765625	0
0.001953125	0
0.00390625	0
0.0078125	0
0.015625	0
0.03125	0
0.0625	0
0.125	0
0.25	0
0.5	1

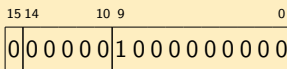
$$0.000030517578125 = 1 \times 2^{-15}$$

$$0.000030517578125 = (0.1)_2 \times 2^{-14}$$

$$s = 0$$

$$\text{exponent} = (00000)_2$$

$$\text{mantissa} = (1000000000)_2$$



N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Example: Decimal to binary₁₆

Present 0.000030517578125 in binary₁₆

0.000030517578125	0
0.00006103515625	0
0.0001220703125	0
0.000244140625	0
0.00048828125	0
0.0009765625	0
0.001953125	0
0.00390625	0
0.0078125	0
0.015625	0
0.03125	0
0.0625	0
0.125	0
0.25	0
0.5	1

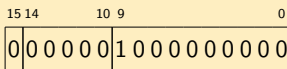
$$0.000030517578125 = 1 \times 2^{-15}$$

$$0.000030517578125 = (0.1)_2 \times 2^{-14}$$

$$s = 0$$

$$\text{exponent} = (00000)_2$$

$$\text{mantissa} = (1000000000)_2$$



$$0.000030517578125 = (0200)_{16} \text{ bin16}$$

N

Peano

Caveperson

Addition

Subtraction

Multiplication

Switching radices

Hardware

Z

Complement

Hardware

Q

Examples

Hardware

Fixed point

BCD

Floating-point

Examples

Minimal Positive

$$2^{-24} = 5.960464477539063 \times 10^{-8} = (0001)_{16 \text{ bin}16}$$

Minimal Non-Negative

$$0 = (0000)_{16 \text{ bin}16}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: binary16 Addition

$$(1.1)_2 \times 2^{-1} + (1.1)_2 \times 2^1 =$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: binary16 Addition

$$(1.1)_2 \times 2^{-1} + (1.1)_2 \times 2^1 =$$
$$(0.011)_2 \times 2^1 + (1.1)_2 \times 2^1 =$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: binary16 Addition

$$\begin{aligned}(1.1)_2 \times 2^{-1} + (1.1)_2 \times 2^1 &= \\ (0.011)_2 \times 2^1 + (1.1)_2 \times 2^1 &= \\ ((0.011)_2 + (1.1)_2) \times 2^1 &= \end{aligned}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: binary16 Addition

$$\begin{aligned}(1.1)_2 \times 2^{-1} + (1.1)_2 \times 2^1 &= \\ (0.011)_2 \times 2^1 + (1.1)_2 \times 2^1 &= \\ ((0.011)_2 + (1.1)_2) \times 2^1 &= \\ (1.111)_2 \times 2^1 &\end{aligned}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: binary16 Addition

$$\begin{aligned}(1.1)_2 \times 2^{-1} + (1.1)_2 \times 2^1 &= \\ (0.011)_2 \times 2^1 + (1.1)_2 \times 2^1 &= \\ ((0.011)_2 + (1.1)_2) \times 2^1 &= \\ (1.111)_2 \times 2^1\end{aligned}$$

$$(1.1)_2 \times 2^0 + (1.1)_2 \times 2^1 =$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: binary16 Addition

$$\begin{aligned}(1.1)_2 \times 2^{-1} + (1.1)_2 \times 2^1 &= \\ (0.011)_2 \times 2^1 + (1.1)_2 \times 2^1 &= \\ ((0.011)_2 + (1.1)_2) \times 2^1 &= \\ (1.111)_2 \times 2^1\end{aligned}$$

$$\begin{aligned}(1.1)_2 \times 2^0 + (1.1)_2 \times 2^1 &= \\ (0.11)_2 \times 2^1 + (1.1)_2 \times 2^1 &= \end{aligned}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: binary16 Addition

$$\begin{aligned}(1.1)_2 \times 2^{-1} + (1.1)_2 \times 2^1 &= \\ (0.011)_2 \times 2^1 + (1.1)_2 \times 2^1 &= \\ ((0.011)_2 + (1.1)_2) \times 2^1 &= \\ (1.111)_2 \times 2^1\end{aligned}$$

$$\begin{aligned}(1.1)_2 \times 2^0 + (1.1)_2 \times 2^1 &= \\ (0.11)_2 \times 2^1 + (1.1)_2 \times 2^1 &= \\ ((0.11)_2 + (1.1)_2) \times 2^1 &= \end{aligned}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: binary16 Addition

$$\begin{aligned}(1.1)_2 \times 2^{-1} + (1.1)_2 \times 2^1 &= \\ (0.011)_2 \times 2^1 + (1.1)_2 \times 2^1 &= \\ ((0.011)_2 + (1.1)_2) \times 2^1 &= \\ (1.111)_2 \times 2^1\end{aligned}$$

$$\begin{aligned}(1.1)_2 \times 2^0 + (1.1)_2 \times 2^1 &= \\ (0.11)_2 \times 2^1 + (1.1)_2 \times 2^1 &= \\ ((0.11)_2 + (1.1)_2) \times 2^1 &= \\ (10.01)_2 \times 2^1 &= \end{aligned}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: binary16 Addition

$$\begin{aligned}(1.1)_2 \times 2^{-1} + (1.1)_2 \times 2^1 &= \\ (0.011)_2 \times 2^1 + (1.1)_2 \times 2^1 &= \\ ((0.011)_2 + (1.1)_2) \times 2^1 &= \\ (1.111)_2 \times 2^1\end{aligned}$$

$$\begin{aligned}(1.1)_2 \times 2^0 + (1.1)_2 \times 2^1 &= \\ (0.11)_2 \times 2^1 + (1.1)_2 \times 2^1 &= \\ ((0.11)_2 + (1.1)_2) \times 2^1 &= \\ (10.01)_2 \times 2^1 &= \\ (1.001)_2 \times 2^2\end{aligned}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: binary16 Addition

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

$$\begin{aligned}(1.1)_2 \times 2^{-1} + (1.1)_2 \times 2^1 &= \\(0.011)_2 \times 2^1 + (1.1)_2 \times 2^1 &= \\((0.011)_2 + (1.1)_2) \times 2^1 &= \\(1.111)_2 \times 2^1 &\end{aligned}$$

$$\begin{aligned}(1.1)_2 \times 2^0 + (1.1)_2 \times 2^1 &= \\(0.11)_2 \times 2^1 + (1.1)_2 \times 2^1 &= \\((0.11)_2 + (1.1)_2) \times 2^1 &= \\(10.01)_2 \times 2^1 &= \\(1.001)_2 \times 2^2 &\end{aligned}$$

Always increase the lower exponent!

Example: binary16 Addition Precision

$$(1.1)_2 \times 2^{-10} + (1.1)_2 \times 2^1 =$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: binary16 Addition Precision

$$(1.1)_2 \times 2^{-10} + (1.1)_2 \times 2^1 =$$
$$(0.000000000011)_2 \times 2^1 + (1.1)_2 \times 2^1 =$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: binary16 Addition Precision

$$\begin{aligned}(1.1)_2 \times 2^{-10} + (1.1)_2 \times 2^1 &= \\ (0.000000000011)_2 \times 2^1 + (1.1)_2 \times 2^1 &= \\ ((0.000000000011)_2 + (1.1)_2) \times 2^1 &= \end{aligned}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: binary16 Addition Precision

$$\begin{aligned}(1.1)_2 \times 2^{-10} + (1.1)_2 \times 2^1 &= \\ (0.000000000011)_2 \times 2^1 + (1.1)_2 \times 2^1 &= \\ ((0.000000000011)_2 + (1.1)_2) \times 2^1 &= \\ (1.100000000011)_2 \times 2^1 &\simeq\end{aligned}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: binary16 Addition Precision

$$\begin{aligned}(1.1)_2 \times 2^{-10} + (1.1)_2 \times 2^1 &= \\(0.000000000011)_2 \times 2^1 + (1.1)_2 \times 2^1 &= \\((0.000000000011)_2 + (1.1)_2) \times 2^1 &= \\(1.100000000011)_2 \times 2^1 &\simeq \\(1.1000000000)_2 \times 2^1 &= \end{aligned}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: binary16 Addition Precision

$$\begin{aligned}(1.1)_2 \times 2^{-10} + (1.1)_2 \times 2^1 &= \\(0.000000000011)_2 \times 2^1 + (1.1)_2 \times 2^1 &= \\((0.000000000011)_2 + (1.1)_2) \times 2^1 &= \\(1.100000000011)_2 \times 2^1 &\simeq \\(1.1000000000)_2 \times 2^1 &= \\(1.1)_2 \times 2^1 &\end{aligned}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: binary16 Multiplication

$$(1.1)_2 \times 2^{-1} \times (1.1)_2 \times 2^1 =$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: binary16 Multiplication

$$\begin{aligned}(1.1)_2 \times 2^{-1} \times (1.1)_2 \times 2^1 &= \\ ((1.1)_2 \times (1.1)_2) \times (2^{-1} \times 2^1) &= \end{aligned}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: binary16 Multiplication

$$\begin{aligned}(1.1)_2 \times 2^{-1} \times (1.1)_2 \times 2^1 &= \\ ((1.1)_2 \times (1.1)_2) \times (2^{-1} \times 2^1) &= \\ (10.01)_2 \times 2^0 &= \end{aligned}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples

Example: binary16 Multiplication

$$\begin{aligned}(1.1)_2 \times 2^{-1} \times (1.1)_2 \times 2^1 &= \\ ((1.1)_2 \times (1.1)_2) \times (2^{-1} \times 2^1) &= \\ (10.01)_2 \times 2^0 &= \\ (1.001)_2 \times 2^1\end{aligned}$$

N

Peano
Caveperson
Addition
Subtraction
Multiplication
Switching radices
Hardware

Z

Complement
Hardware

Q

Examples
Hardware
Fixed point
BCD
Floating-point
Examples