Carmi Merimovich

The Academic College of Tel-Aviv

January 29, 2025

• Introducing boolean algebras

• Present examples for their usage

Boolean Algebra

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Definition

The structure $\langle \mathbb{B}, 0, 1, +, \cdot, \bar{} \rangle$, where

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Definition

The structure $\langle \mathbb{B}, 0, 1, +, \cdot, \bar{} \rangle$, where

• \mathbb{B} is a set.

Boolean Algebra

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Definition

The structure $\langle \mathbb{B}, 0, 1, +, \cdot, \bar{} \rangle$, where

- \mathbb{B} is a set,
- 0 and 1 are constants (presumbly, not equal),

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- is a unary operator (1-ary functions),

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- \mathbb{B} is a set,
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- ⁻ is a unary operator (1-ary functions),

is a boolean algebra if the following hold for each x, y, z,

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- B is a set.
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- is a unary operator (1-ary functions),

is a boolean algebra if the following hold for each x, y, z,

$$x + 0 = x \qquad x \cdot 1 = x$$

$$x \cdot 1 = x$$

Comutativity:
$$x + y = y + x$$
 $x \cdot y = y \cdot x$

Boolean Algebra

Definition

The structure $\langle \mathbb{B}, 0, 1, +, \cdot, \bar{} \rangle$, where

- \mathbb{B} is a set,
- 0 and 1 are constants (presumbly, not equal),
- \bullet + and \cdot are binary operators (i.e., 2-ary functions),
- ⁻ is a unary operator (1-ary functions),

is a boolean algebra if the following hold for each x, y, z,

Neutrality: x + 0 = x $x \cdot 1 = x$

Comutativity: x + y = y + x $x \cdot y = y \cdot x$

Associativity: $(x+y)+z=(x\cdot y)\cdot z=$

x + (y + z) $x \cdot (y \cdot z)$

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Definition

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- B is a set.
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- \bullet + and \cdot are binary operators (i.e., 2-ary functions),
- is a unary operator (1-ary functions),

is a boolean algebra if the following hold for each x, y, z,

Neutrality: x + 0 = x $x \cdot 1 = x$

Comutativity: x + y = y + x $x \cdot y = y \cdot x$

(x+y)+z= $(x \cdot y) \cdot z =$ Associativity:

$$x + (y + z)$$
 $x \cdot (y \cdot z)$

 $x \cdot (y+z) =$ $x + (y \cdot z) =$ Distributivity:

$$x \cdot y + x \cdot z$$
 $(x+y) \cdot (x+z)$

Boolean Algebra

- B is a set,
- 0 and 1 are constants (presumbly, not equal),
- + and · are binary operators (i.e., 2-ary functions),
- ⁻ is a unary operator (1-ary functions),

is a boolean algebra if the following hold for each $x,\ y,\ z,$

Neutrality: x + 0 = x $x \cdot 1 = x$ Comutativity: x + y = y + x $x \cdot y = y \cdot x$

Associativity: $(x+y)+z=(x\cdot y)\cdot z=$

x + (y + z) $x \cdot (y \cdot z)$

Distributivity: $x \cdot (y+z) = x+(y \cdot z) =$

 $x \cdot y + x \cdot z$ $(x+y) \cdot (x+z)$

Complement: $x + \bar{x} = 1$ $x \cdot \bar{x} = 0$ unique?

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If xy = 0, x + y = 1, xz = 0 and x + z = 1 then y = z.

Proof.

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If
$$xy=0$$
, $x+y=1$, $xz=0$ and $x+z=1$ then $y=z$.

Proof.

$$y = y \cdot 1 =$$

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If
$$xy = 0$$
, $x + y = 1$, $xz = 0$ and $x + z = 1$ then $y = z$.

Proof.

$$y = y \cdot 1 =$$
$$= y \cdot (x + z) =$$

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If
$$xy=0$$
, $x+y=1$, $xz=0$ and $x+z=1$ then $y=z$.

Proof.

$$y = y \cdot 1 =$$

$$= y \cdot (x + z) =$$

$$= y \cdot x + y \cdot z =$$

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, $x+y=1$, $xz=0$ and $x+z=1$ then $y=z$.

Proof.

$$y = y \cdot 1 =$$

$$= y \cdot (x + z) =$$

$$= y \cdot x + y \cdot z =$$

$$= 0 + y \cdot z =$$

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Proof.

$$y = y \cdot 1 =$$

$$= y \cdot (x + z) =$$

$$= y \cdot x + y \cdot z =$$

$$= 0 + y \cdot z =$$

$$= x \cdot z + y \cdot z =$$

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If
$$xy=0$$
, $x+y=1$, $xz=0$ and $x+z=1$ then $y=z$.

Proof.

$$y = y \cdot 1 =$$
 $= y \cdot (x + z) =$
 $= y \cdot x + y \cdot z =$
 $= 0 + y \cdot z =$
 $= x \cdot z + y \cdot z =$
 $= z \cdot (x + y) =$

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If
$$xy=0$$
, $x+y=1$, $xz=0$ and $x+z=1$ then $y=z$.

Proof.

$$y = y \cdot 1 =$$
 $= y \cdot (x + z) =$
 $= y \cdot x + y \cdot z =$
 $= 0 + y \cdot z =$
 $= x \cdot z + y \cdot z =$
 $= z \cdot (x + y) =$
 $= z \cdot 1 =$

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If
$$xy=0$$
, $x+y=1$, $xz=0$ and $x+z=1$ then $y=z$.

Proof.

$$y = y \cdot 1 =$$

$$= y \cdot (x + z) =$$

$$= y \cdot x + y \cdot z =$$

$$= 0 + y \cdot z =$$

$$= x \cdot z + y \cdot z =$$

$$= z \cdot (x + y) =$$

$$= z \cdot 1 =$$

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Lemma

Assume $\mathbb B$ is a boolean algebra. Then the set of constants $\{0,1\}$ is closed under the operators +, \cdot and $\bar{}$. (i.e., $\{0,1\}$ is a boolean subalgebra of $\mathbb B$.)

Proof.

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Lemma

Assume $\mathbb B$ is a boolean algebra. Then the set of constants $\{0,1\}$ is closed under the operators +, \cdot and $\bar{}$. (i.e., $\{0,1\}$ is a boolean subalgebra of $\mathbb B$.)

Proof.

1. $\bar{}$: By the relevant neutralities of 0 and 1 we have 0+1=1 and $0\cdot 1=0$. Thus $\bar{0}=1$ and $\bar{1}=0$.

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Proof.

- 1. $\bar{}$: By the relevant neutralities of 0 and 1 we have 0+1=1 and $0\cdot 1=0$. Thus $\bar{0}=1$ and $\bar{1}=0$.
- 2. \cdot : By the neutrality of 1 we get $1\cdot 0=0\cdot 1=0$ and $1\cdot 1=1$. For $0\cdot 0$ we work as follows.

$$0 \cdot 0 = 0 \cdot 0 + 0 = 0 \cdot 0 + (0 \cdot 1) = 0 \cdot (0 + 1) = 0 \cdot 1 = 0.$$

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Lemma

Assume $\mathbb B$ is a boolean algebra. Then the set of constants $\{0,1\}$ is closed under the operators +, \cdot and $\bar{}$. (i.e., $\{0,1\}$ is a boolean subalgebra of $\mathbb B$.)

Proof.

- 1. $\bar{}$: By the relevant neutralities of 0 and 1 we have 0+1=1 and $0\cdot 1=0$. Thus $\bar{0}=1$ and $\bar{1}=0$.
- 2. \cdot : By the neutrality of 1 we get $1\cdot 0=0\cdot 1=0$ and $1\cdot 1=1.$ For $0\cdot 0$ we work as follows.

$$0 \cdot 0 = 0 \cdot 0 + 0 = 0 \cdot 0 + (0 \cdot 1) = 0 \cdot (0 + 1) = 0 \cdot 1 = 0.$$

3. +: By the neutrality of 0 we get 1+0=0+1=1 and 0+0=0. For 1+1 we work as follows.

$$1+1 = (1+1)\cdot 1 = (1+1)\cdot (1+0) = 1+(1\cdot 0) = 1+0 = 1.$$

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Truth table form of the lemma

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x	\bar{x}
0	1
1	0

x	y	$x \cdot y$
0	0	0
0	1	0
1	0	0
1	1	1

\boldsymbol{x}	y	x + y
0	0	0
0	1	1
1	0	1
1	1	1

- If there is a boolean algebra then
 - its 0 and 1 follow the above tables
- No boolean algebra has been spotted as of yet!

The 2-Valued boolean algebra

Let us take the truth tables above as a **definition**.

Associativity of ·

x	y	z	$x \cdot y$	$(x \cdot y) \cdot z$	$y \cdot z$	$x \cdot (y \cdot z)$
0	0	0				
0	0	1				
0	1	0				
0	1	1				
1	0	0				
1	0	1				
1	1	0				
1	1	1				

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Associativity of ·

x	y	z	$x \cdot y$	$(x \cdot y) \cdot z$	$y \cdot z$	$x \cdot (y \cdot z)$
0	0	0	0			
0	0	1	0			
0	1	0	0			
0	1	1	0			
1	0	0	0			
1	0	1	0			
1	1	0	1			
1	1	1	1			

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Associativity of ·

x	y	z	$x \cdot y$	$(x \cdot y) \cdot z$	$y \cdot z$	$x \cdot (y \cdot z)$
0	0	0	0	0		
0	0	1	0	0		
0	1	0	0	0		
0	1	1	0	0		
1	0	0	0	0		
1	0	1	0	0		
1	1	0	1	0		
1	1	1	1	1		

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Associativity of ·

x	y	z	$x \cdot y$	$(x \cdot y) \cdot z$	$y \cdot z$	$x \cdot (y \cdot z)$
0	0	0	0	0	0	
0	0	1	0	0	0	
0	1	0	0	0	0	
0	1	1	0	0	1	
1	0	0	0	0	0	
1	0	1	0	0	0	
1	1	0	1	0	0	
1	1	1	1	1	1	

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Associativity of ·

\boldsymbol{x}	y	z	$x \cdot y$	$(x \cdot y) \cdot z$	$y \cdot z$	$x \cdot (y \cdot z)$
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	0	0	0
0	1	1	0	0	1	0
1	0	0	0	0	0	0
1	0	1	0	0	0	0
1	1	0	1	0	0	0
1	1	1	1	1	1	1

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\boldsymbol{x}	y	z	$x \cdot y$	$(x \cdot y) \cdot z$	$y \cdot z$	$x \cdot (y \cdot z)$
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	0	0	0
0	1	1	0	0	1	0
1	0	0	0	0	0	0
1	0	1	0	0	0	0
1	1	0	1	0	0	0
1	1	1	1	1	1	1

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x	y	z	$x \cdot y$	$(x \cdot y) \cdot z$	$y \cdot z$	$x \cdot (y \cdot z)$
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	0	0	0
0	1	1	0	0	1	0
1	0	0	0	0	0	0
1	0	1	0	0	0	0
1	1	0	1	0	0	0
1	1	1	1	1	1	1

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Associativity of +

x	y	z	x+y	(x+y)+z	y+z	x + (y+z)
0	0	0				
0	0	1				
0	1	0				
0	1	1				
1	0	0				
1	0	1				
1	1	0				
1	1	1				

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x	y	z	x + y	(x+y)+z	y+z	x + (y + z)
0	0	0	0			
0	0	1	0			
0	1	0	1			
0	1	1	1			
1	0	0	1			
1	0	1	1			
1	1	0	1			
1	1	1	1			

x	y	z	x+y	(x+y)+z	y + z	x + (y + z)
0	0	0	0	0		
0	0	1	0	1		
0	1	0	1	1		
0	1	1	1	1		
1	0	0	1	1		
1	0	1	1	1		
1	1	0	1	1		
1	1	1	1	1		

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x	y	z	x+y	(x+y)+z	y+z	x + (y+z)
0	0	0	0	0	0	
0	0	1	0	1	1	
0	1	0	1	1	1	
0	1	1	1	1	1	
1	0	0	1	1	0	
1	0	1	1	1	1	
1	1	0	1	1	1	
1	1	1	1	1	1	

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x	y	z	x+y	(x+y)+z	y + z	x + (y+z)
0	0	0	0	0	0	0
0	0	1	0	1	1	1
0	1	0	1	1	1	1
0	1	1	1	1	1	1
1	0	0	1	1	0	1
1	0	1	1	1	1	1
1	1	0	1	1	1	1
1	1	1	1	1	1	1

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x	y	z	x+y	(x+y)+z	y + z	x + (y+z)
0	0	0	0	0	0	0
0	0	1	0	1	1	1
0	1	0	1	1	1	1
0	1	1	1	1	1	1
1	0	0	1	1	0	1
1	0	1	1	1	1	1
1	1	0	1	1	1	1
1	1	1	1	1	1	1

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Associativity of +

x	y	z	x+y	(x+y)+z	y+z	x + (y+z)
0	0	0	0	0	0	0
0	0	1	0	1	1	1
0	1	0	1	1	1	1
0	1	1	1	1	1	1
1	0	0	1	1	0	1
1	0	1	1	1	1	1
1	1	0	1	1	1	1
1	1	1	1	1	1	1

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ield of Sets

\boldsymbol{x}	y	z	y+z	$x \cdot (y+z)$	$x \cdot y$	$x \cdot z$	$x \cdot y + x \cdot z$
0	0	0					
0	0	1					
0	1	0					
0	1	1					
1	0	0					
1	0	1					
1	1	0					
1	1	1					

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Distributivity of ⋅ over +

\boldsymbol{x}	y	z	y+z	$x \cdot (y+z)$	$x \cdot y$	$x \cdot z$	$x \cdot y + x \cdot z$
0	0	0	0				
0	0	1	1				
0	1	0	1				
0	1	1	1				
1	0	0	0				
1	0	1	1				
1	1	0	1				
1	1	1	1				

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\boldsymbol{x}	y	z	y+z	$x \cdot (y+z)$	$x \cdot y$	$x \cdot z$	$x \cdot y + x \cdot z$
0	0	0	0	0			
0	0	1	1	0			
0	1	0	1	0			
0	1	1	1	0			
1	0	0	0	0			
1	0	1	1	1			
1	1	0	1	1			
1	1	1	1	1			

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\boldsymbol{x}	y	z	y+z	$x \cdot (y+z)$	$x \cdot y$	$x \cdot z$	$x \cdot y + x \cdot z$
0	0	0	0	0	0		
0	0	1	1	0	0		
0	1	0	1	0	0		
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0	0	1	1	0	0	0	
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1	0	0	0	0	0	0	0
1	0	1	1	1	0	1	1
1	1	0	1	1	1	0	1
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0	0	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	1	1	0	1	1
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\boldsymbol{x}	y	z	y+z	$x \cdot (y+z)$	$x \cdot y$	$x \cdot z$	$x \cdot y + x \cdot z$
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0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0
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x	y	z	$y \cdot z$	$x + (y \cdot z) x + y x + z (x + y) \cdot (x + z)$
0	0	0	0	
0	0	1	0	
0	1	0	0	
0	1	1	1	
1	0	0	0	
1	0	1	0	
1	1	0	0	
1	1	1	1	

x y z	$y \cdot z$	$x + (y \cdot z)$	x + y	x+z	$(x+y)\cdot(x+z)$
0 0 0	0	0			
0 0 1	0	0			
0 1 0	0	0			
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x y z	$y \cdot z$	$x + (y \cdot z)$	x + y	$x+z (x+y) \cdot (x+z)$
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x y z	$y \cdot z$	$x + (y \cdot z)$	x + y	x + z	$(x+y)\cdot(x+z)$
0 0 0	0	0	0	0	
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0 1 0	0	0	1	0	
0 1 1	1	1	1	1	
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x y z	$y \cdot z$	$x + (y \cdot z)$	x + y	x + z	$(x+y)\cdot(x+z)$
0 0 0	0	0	0	0	0
0 0 1	0	0	0	1	0
0 1 0	0	0	1	0	0
0 1 1	1	1	1	1	1
1 0 0	0	1	1	1	1
1 0 1	0	1	1	1	1
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0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0
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x y z	$y \cdot z$	$x + (y \cdot z)$	x + y	x + z	$(x+y)\cdot(x+z)$
0 0 0	0	0	0	0	0
0 0 1	0	0	0	1	0
0 1 0	0	0	1	0	0
0 1 1	1	1	1	1	1
1 0 0	0	1	1	1	1
1 0 1	0	1	1	1	1
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2-Valued Booean Algebra

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Corollary

 $\langle \{0,1\},0,1,+,\cdot,\bar{}\rangle$ is a boolean algebra.

Digital Design

Only the 2-valued boolean algebra is used

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High school conventions

- 1. The '·' symbol can be dropped, i.e., $xy = x \cdot y$.
- 2. '·' takes precedence over '+', e.g., $x+yz=x+(y\cdot z)$.

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Definition

A function $f: \mathbb{B}^n \to \mathbb{B}^m$ is called a boolean function.

Note

- 1. is a 1-ary function
- 2. Both \cdot and + are 2-ary functions

Exclusive-or, xor (\oplus)

This is a function we all know and we use infix notation for it.

x	y	$x \oplus y$
0	0	0
0	1	1
1	0	1
1	1	0

Boolean Functions

13 / 68

- There might be functions without a defining formula
- This is the situation with functions $\mathbb{R} \to \mathbb{R}$

Definition

- A boolean function f is definable if it falls into one of the following cases:
 - ightharpoonup f = 0 or f = 1 or $f = x_i$.
 - $ightharpoonup f = (\bar{g})$, where g is a definable
 - ▶ $f = (f_0 + f_1)$ where both f_0 and f_1 are definable
 - $ightharpoonup f = (f_0 \cdot f_1)$ where both f_0 and f_1 are definable.
 - $f = g(h_0, \dots, h_{n-1})$ where g, h_0, \dots, h_{n-1} are definable

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Definable functions have a truth table representation

			f(x, y, z) = x + yz
\boldsymbol{x}	y	z	$y \cdot z \qquad x + y \cdot z$
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

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			f(x, y)	y,z) = x + yz
x	y	z	$y \cdot z$	$x + y \cdot z$
0	0	0	0	
0	0	1	0	
0	1	0	0	
0	1	1	1	
1	0	0	0	
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			f(x, y, z) = x + yz		
x	y	z	$y \cdot z$	$x + y \cdot z$	
0	0	0	0	0	
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			f(x, y, z) = x + yz		
x	y	z	$y \cdot z$	$x + y \cdot z$	
0	0	0	0	0	
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Field of Sets

- Definable boolean functions can be realized in hardware
- Boolean functions can realize useful operations
- We need some motiviation!

(Carmi) Lecture 4 reached here

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Problem

The seven segment component has seven light segments as follows:

$$\begin{bmatrix} f & g \\ e & g \\ d & c \end{bmatrix}$$

$$b = c = 1, a = d = e = f = g = 0$$

Devise a boolean formula accepting a decimal digit and showing it on the seven segment

The function form

- The decimal digits will be coded in binary
- Each of the segments needs its own line
- $f = \langle g, f, e, d, c, b, a \rangle : \mathbb{B}^4 \to \mathbb{B}^7_{0/68}$

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Seven Segment (Symbols to Boolean)

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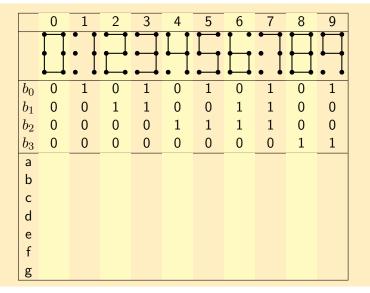
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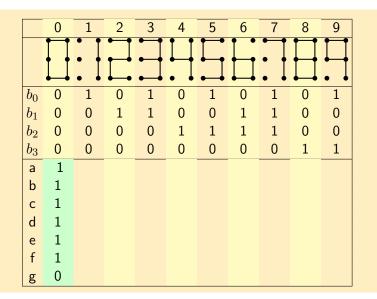


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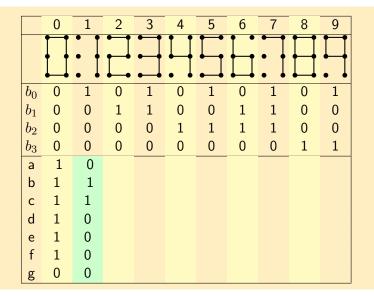
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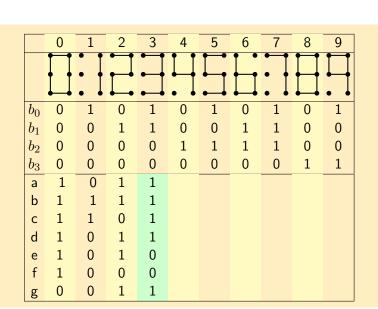
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b_1	0	0	1	1	0	0	1	1	0	0
b_2	0	0	0	0	1	1	1	1	0	0
b_3	0	0	0	0	0	0	0	0	1	1
а	1	0	1							
b	1	1	1							
С	1	1	0							
d	1	0	1							
е	1	0	1							
f	1	0	0							
g	0	0	1							

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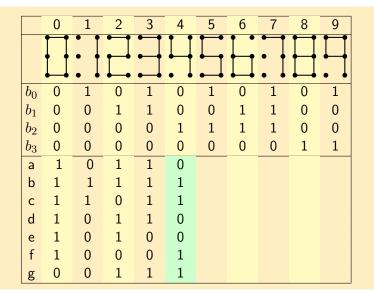
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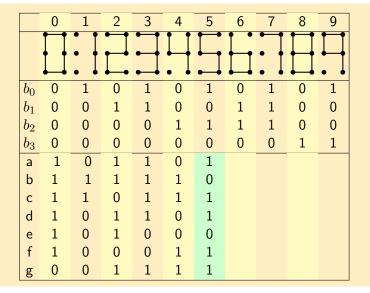
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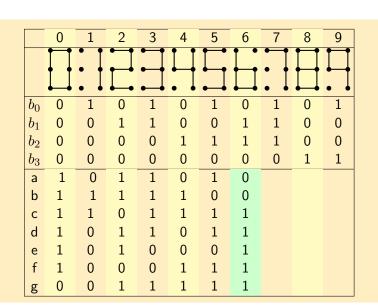
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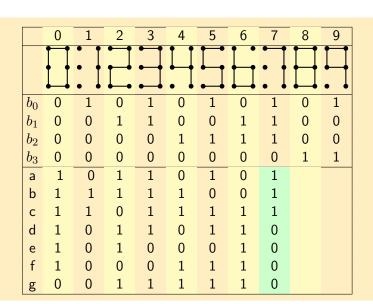
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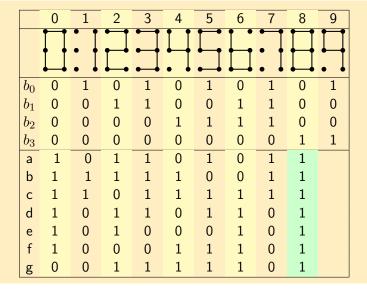
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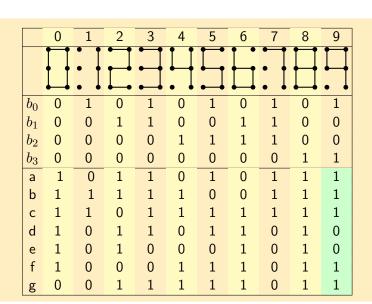
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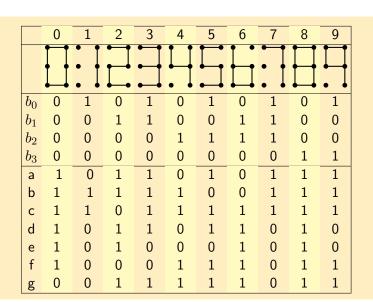
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Stone's Theoren

- There are four inputs b_3, b_2, b_1, b_0
- The target functions depend on the decimal digits
- (As is evident by the previous slide)
- ullet Temporary functions m_0,\ldots,m_9 might be easier on us

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b_3	b_2	b_1	b_0	m_0	m_1	m_2	m_3	m_4	m_5	m_6	m_7	m_8	m_9
0	0	0	0										
0	0	0	1										
0	0	1	0										
0	0	1	1										
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b_3	b_2	b_1	b_0	m_0	m_1	m_2	m_3	m_4	m_5	m_6	m_7	m_8	m_9
0	0	0	0	1									
0	0	0	1	0									
0	0	1	0	0									
0	0	1	1	0									
0	1	0	0	0									
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0	1	1	0	0									
0	1	1	1	0									
1	0	0	0	0									
1	0	0	1	0									

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b_3	b_2	b_1	b_0	m_0	m_1	m_2	m_3	m_4	m_5	m_6	m_7	m_8	m_9
0	0	0	0	1	0								
0	0	0	1	0	1								
0	0	1	0	0	0								
0	0	1	1	0	0								
0	1	0	0	0	0								
0	1	0	1	0	0								
0	1	1	0	0	0								
0	1	1	1	0	0								
1	0	0	0	0	0								
1	0	0	1	0	0								

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b_3	b_2	b_1	b_0	m_0	m_1	m_2	m_3	m_4	m_5	m_6	m_7	m_8	m_9
0	0	0	0	1	0	0							
0	0	0	1	0	1	0							
0	0	1	0	0	0	1							
0	0	1	1	0	0	0							
0	1	0	0	0	0	0							
0	1	0	1	0	0	0							
0	1	1	0	0	0	0							
0	1	1	1	0	0	0							
1	0	0	0	0	0	0							
1	0	0	1	0	0	0							

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b_3	b_2	b_1	b_0	m_0	m_1	m_2	m_3	m_4	m_5	m_6	m_7	m_8	m_9
0	0	0	0	1	0	0	0						
0	0	0	1	0	1	0	0						
0	0	1	0	0	0	1	0						
0	0	1	1	0	0	0	1						
0	1	0	0	0	0	0	0						
0	1	0	1	0	0	0	0						
0	1	1	0	0	0	0	0						
0	1	1	1	0	0	0	0						
1	0	0	0	0	0	0	0						
1	0	0	1	0	0	0	0						

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b_3	b_2	b_1	b_0	m_0	m_1	m_2	m_3	m_4	m_5	m_6	m_7	m_8	m_9
0	0	0	0	1	0	0	0	0					
0	0	0	1	0	1	0	0	0					
0	0	1	0	0	0	1	0	0					
0	0	1	1	0	0	0	1	0					
0	1	0	0	0	0	0	0	1					
0	1	0	1	0	0	0	0	0					
0	1	1	0	0	0	0	0	0					
0	1	1	1	0	0	0	0	0					
1	0	0	0	0	0	0	0	0					
1	0	0	1	0	0	0	0	0					

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l	b_3	b_2	b_1	b_0	m_0	m_1	m_2	m_3	m_4	m_5	m_6	m_7	m_8	m_9
	0	0	0	0	1	0	0	0	0	0				
	0	0	0	1	0	1	0	0	0	0				
	0	0	1	0	0	0	1	0	0	0				
	0	0	1	1	0	0	0	1	0	0				
	0	1	0	0	0	0	0	0	1	0				
	0	1	0	1	0	0	0	0	0	1				
	0	1	1	0	0	0	0	0	0	0				
	0	1	1	1	0	0	0	0	0	0				
	1	0	0	0	0	0	0	0	0	0				
	1	0	0	1	0	0	0	0	0	0				

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b_3	b_2	b_1	b_0	m_0	m_1	m_2	m_3	m_4	m_5	m_6	m_7	m_8	m_9
0	0	0	0	1	0	0	0	0	0	0			
0	0	0	1	0	1	0	0	0	0	0			
0	0	1	0	0	0	1	0	0	0	0			
0	0	1	1	0	0	0	1	0	0	0			
0	1	0	0	0	0	0	0	1	0	0			
0	1	0	1	0	0	0	0	0	1	0			
0	1	1	0	0	0	0	0	0	0	1			
0	1	1	1	0	0	0	0	0	0	0			
1	0	0	0	0	0	0	0	0	0	0			
1	0	0	1	0	0	0	0	0	0	0			

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b_3	b_2	b_1	b_0	m_0	m_1	m_2	m_3	m_4	m_5	m_6	m_7	m_8	m_9
0	0	0	0	1	0	0	0	0	0	0	0		
0	0	0	1	0	1	0	0	0	0	0	0		
0	0	1	0	0	0	1	0	0	0	0	0		
0	0	1	1	0	0	0	1	0	0	0	0		
0	1	0	0	0	0	0	0	1	0	0	0		
0	1	0	1	0	0	0	0	0	1	0	0		
0	1	1	0	0	0	0	0	0	0	1	0		
0	1	1	1	0	0	0	0	0	0	0	1		
1	0	0	0	0	0	0	0	0	0	0	0		
1	0	0	1	0	0	0	0	0	0	0	0		

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b_3	b_2	b_1	b_0	m_0	m_1	m_2	m_3	m_4	m_5	m_6	m_7	m_8	m_9
0	0	0	0	1	0	0	0	0	0	0	0	0	
0	0	0	1	0	1	0	0	0	0	0	0	0	
0	0	1	0	0	0	1	0	0	0	0	0	0	
0	0	1	1	0	0	0	1	0	0	0	0	0	
0	1	0	0	0	0	0	0	1	0	0	0	0	
0	1	0	1	0	0	0	0	0	1	0	0	0	
0	1	1	0	0	0	0	0	0	0	1	0	0	
0	1	1	1	0	0	0	0	0	0	0	1	0	
1	0	0	0	0	0	0	0	0	0	0	0	1	
1	0	0	1	0	0	0	0	0	0	0	0	0	

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b_3	b_2	b_1	b_0	m_0	m_1	m_2	m_3	m_4	m_5	m_6	m_7	m_8	m_9
0	0	0	0	1	0	0	0	0	0	0	0	0	0
0	0	0	1	0	1	0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0	0	0	0	0	0	0
0	0	1	1	0	0	0	1	0	0	0	0	0	0
0	1	0	0	0	0	0	0	1	0	0	0	0	0
0	1	0	1	0	0	0	0	0	1	0	0	0	0
0	1	1	0	0	0	0	0	0	0	1	0	0	0
0	1	1	1	0	0	0	0	0	0	0	1	0	0
1	0	0	0	0	0	0	0	0	0	0	0	1	0
1	0	0	1	0	0	0	0	0	0	0	0	0	1

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b_3	b_2	b_1	b_0	m_0	m_1	m_2	m_3	m_4	m_5	m_6	m_7	m_8	m_9
0	0	0	0	1	0	0	0	0	0	0	0	0	0
0	0	0	1	0	1	0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0	0	0	0	0	0	0
0	0	1	1	0	0	0	1	0	0	0	0	0	0
0	1	0	0	0	0	0	0	1	0	0	0	0	0
0	1	0	1	0	0	0	0	0	1	0	0	0	0
0	1	1	0	0	0	0	0	0	0	1	0	0	0
0	1	1	1	0	0	0	0	0	0	0	1	0	0
1	0	0	0	0	0	0	0	0	0	0	0	1	0
1	0	0	1	0	0	0	0	0	0	0	0	0	1

b_3	b_2	b_1	b_0	m_0	m_1	m_2	m_3	m_4	m_5	m_6	m_7	m_8	m_9
0	0	0	0	1	0	0	0	0	0	0	0	0	0
0	0	0	1	0	1	0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0	0	0	0	0	0	0
0	0	1	1	0	0	0	1	0	0	0	0	0	0
0	1	0	0	0	0	0	0	1	0	0	0	0	0
0	1	0	1	0	0	0	0	0	1	0	0	0	0
0	1	1	0	0	0	0	0	0	0	1	0	0	0
0	1	1	1	0	0	0	0	0	0	0	1	0	0
1	0	0	0	0	0	0	0	0	0	0	0	1	0
1	0	0	1	0	0	0	0	0	0	0	0	0	1

$$\begin{split} m_0 &= \bar{b}_3 \bar{b}_2 \bar{b}_1 \bar{b}_0 & m_3 = \bar{b}_3 \bar{b}_2 b_1 b_0 & m_6 = \bar{b}_3 b_2 b_1 \bar{b}_0 & m_9 = b_3 \bar{b}_2 \bar{b}_1 b_0 \\ m_1 &= \bar{b}_3 \bar{b}_2 \bar{b}_1 b_0 & m_4 = \bar{b}_3 b_2 \bar{b}_1 \bar{b}_0 & m_7 = \bar{b}_3 b_2 b_1 b_0 \\ m_2 &= \bar{b}_3 \bar{b}_2 b_1 \bar{b}_0 & m_5 = \bar{b}_3 b_2 \bar{b}_1 b_0 & m_8 = b_3 \bar{b}_2 \bar{b}_1 \bar{b}_0 \end{split}$$

Boolean Algebra

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Seven Segment (calculating a)

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m_0	m_1	m_2	m_3	m_4	m_5	m_6	m_7	m_8	m_9	a
1	0	0	0	0	0	0	0	0	0	1
0	1	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	1
0	0	0	1	0	0	0	0	0	0	1
0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0	1
0	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	1	0	0	1
0	0	0	0	0	0	0	0	1	0	1
0	0	0	0	0	0	0	0	0	1	1

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	m_0	m_1	m_2	m_3	m_4	m_5	m_6	m_7	m_8	m_9	a
	1	0	0	0	0	0	0	0	0	0	1
	0	1	0	0	0	0	0	0	0	0	0
	0	0	1	0	0	0	0	0	0	0	1
	0	0	0	1	0	0	0	0	0	0	1
	0	0	0	0	1	0	0	0	0	0	0
	0	0	0	0	0	1	0	0	0	0	1
	0	0	0	0	0	0	1	0	0	0	0
I	0	0	0	0	0	0	0	1	0	0	1
	0	0	0	0	0	0	0	0	1	0	1
	0	0	0	0	0	0	0	0	0	1	1

$$a = m_0 + m_2 + m_3 + m_5 + m_7 + m_8 + m_9$$

Seven Segment (calculating a)

m_0	m_1	m_2	m_3	m_4	m_5	m_6	m_7	m_8	m_9	a
1	0	0	0	0	0	0	0	0	0	1
0	1	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	1
0	0	0	1	0	0	0	0	0	0	1
0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0	1
0	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	1	0	0	1
0	0	0	0	0	0	0	0	1	0	1
0	0	0	0	0	0	0	0	0	1	1

$$a = m_0 + m_2 + m_3 + m_5 + m_7 + m_8 + m_9$$

 $a = \bar{m}_1 \bar{m}_4 \bar{m}_6$

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Seven segment

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Seven Segment (calculating b)

m_0	m_1	m_2	m_3	m_4	m_5	m_6	m_7	m_8	m_9	b
1	0	0	0	0	0	0	0	0	0	1
0	1	0	0	0	0	0	0	0	0	1
0	0	1	0	0	0	0	0	0	0	1
0	0	0	1	0	0	0	0	0	0	1
0	0	0	0	1	0	0	0	0	0	1
0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	1	0	0	1
0	0	0	0	0	0	0	0	1	0	1
0	0	0	0	0	0	0	0	0	1	1

$$b = m_0 + m_1 + m_2 + m_3 + m_4 + m_7 + m_8 + m_9$$
$$b = \bar{m}_5 \bar{m}_6$$

Boolean Algebra

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Boolean Functions

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Seven Segment (calculating c)

m_0	m_1	m_2	m_3	m_4	m_5	m_6	m_7	m_8	m_9	c
1	0	0	0	0	0	0	0	0	0	1
0	1	0	0	0	0	0	0	0	0	1
0	0	1	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	1
0	0	0	0	1	0	0	0	0	0	1
0	0	0	0	0	1	0	0	0	0	1
0	0	0	0	0	0	1	0	0	0	1
0	0	0	0	0	0	0	1	0	0	1
0	0	0	0	0	0	0	0	1	0	1
0	0	0	0	0	0	0	0	0	1	1

$$c = m_0 + m_1 + m_3 + m_4 + m_5 + m_6 + m_7 + m_8 + m_9$$
$$c = \bar{m}_2$$

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Seven Segment (calculating d)

m_0	m_1	m_2	m_3	m_4	m_5	m_6	m_7	m_8	m_9	d
1	0	0	0	0	0	0	0	0	0	1
0	1	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	1
0	0	0	1	0	0	0	0	0	0	1
0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0	1
0	0	0	0	0	0	1	0	0	0	1
0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	1	0	1
0	0	0	0	0	0	0	0	0	1	0

$$d = m_0 + m_2 + m_3 + m_5 + m_6 + m_8$$
$$d = \bar{m}_1 \bar{m}_4 \bar{m}_7 \bar{m}_9$$

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m_0	m_1	m_2	m_3	m_4	m_5	m_6	m_7	m_8	d_9	e
1	0	0	0	0	0	0	0	0	0	1
0	1	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	1
0	0	0	1	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	1
0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	1	0	1
0	0	0	0	0	0	0	0	0	1	0

$$e = m_0 + m_2 + m_6 + m_8$$

 $e = \bar{m}_1 \bar{m}_3 \bar{m}_4 \bar{m}_5 \bar{m}_7 \bar{m}_9$

Seven Segment (calculating f)

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m_0	m_1	m_2	m_3	m_4	m_5	m_6	m_7	m_8	d_9	f
1	0	0	0	0	0	0	0	0	0	1
0	1	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0	1
0	0	0	0	0	1	0	0	0	0	1
0	0	0	0	0	0	1	0	0	0	1
0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	1	0	1
0	0	0	0	0	0	0	0	0	1	1

$$f = m_0 + m_4 + m_5 + m_6 + m_8 + m_9$$

$$f = \bar{m}_1 \bar{m}_2 \bar{m}_3 \bar{m}_7$$

Seven Segment (calculating g)

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	m_0	m_1	m_2	m_3	m_4	m_5	m_6	m_7	m_8	d_9	g
	1	0	0	0	0	0	0	0	0	0	0
	0	1	0	0	0	0	0	0	0	0	0
	0	0	1	0	0	0	0	0	0	0	1
	0	0	0	1	0	0	0	0	0	0	1
	0	0	0	0	1	0	0	0	0	0	1
	0	0	0	0	0	1	0	0	0	0	1
	0	0	0	0	0	0	1	0	0	0	1
	0	0	0	0	0	0	0	1	0	0	0
	0	0	0	0	0	0	0	0	1	0	1
	0	0	0	0	0	0	0	0	0	1	1

$$g = m_2 + m_3 + m_4 + m_5 + m_6 + m_8 + m_9$$

 $g = \bar{m}_0 \bar{m}_1 \bar{m}_7$

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- Boolean Algebra
- Minimal BA
- Convention
- Boolean Function
- Definable functions
- Seven segment
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- ofinab
- Jnwinding
- Addition
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- n-Bits Bina
- ield of Sets
- Stone's Theorem

- Lots of work
- For illegal input the output is garbage
- No mathematical laws used in the table construction
- Here, we have no way but to use truth tables
- In this case reading formula from the table is easy

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Problem

Devise a boolean formula for computing $n \bmod 3$ for $0 \le n \le 15$

The function form

- We use binary coding
- 4-bits input
- 2-bits output
- Thus the function is of the form $f = \langle f_1, f_0 \rangle : \mathbb{B}^4 \to \mathbb{B}^2$

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Boolean Functions

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	Decimal			Bin	ary		
n	$n \mod 3$	n_3	n_2	n_1	n_0	f_1	f_0
0		0	0	0	0		
1		0	0	0	1		
2 3		0	0	1	0		
3		0	0	1	1		
4		0	1	0	0		
5		0	1	0	1		
6		0	1	1	0		
8		0	1	1	1		
		1	0	0	0		
9		1	0	0	1		
10		1	0	1	0		
11		1	0	1	1		
12		1	1	0	0		
13		1	1	0	1		
14		1	1	1	0		
15		1	1	1	1		

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	Decimal			Bin	ary		
n	$n \mod 3$	n_3	n_2	n_1	n_0	f_1	f_0
0	0	0	0	0	0	0	0
1		0	0	0	1		
1 2 3		0	0	1	0		
3		0	0	1	1		
4		0	1	0	0		
5		0	1	0	1		
6		0	1	1	0		
7		0	1	1	1		
8		1	0	0	0		
9		1	0	0	1		
10		1	0	1	0		
11		1	0	1	1		
12		1	1	0	0		
13		1	1	0	1		
14		1	1	1	0		
15		1	1	1	1		

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	Decimal			Bin	ary		
n	$n \mod 3$	n_3	n_2	n_1	n_0	f_1	f_0
0	0	0	0	0	0	0	0
1	1	0	0	0	1	0	1
$\begin{vmatrix} 2\\ 3 \end{vmatrix}$		0	0	1	0		
3		0	0	1	1		
4		0	1	0	0		
5		0	1	0	1		
6		0	1	1	0		
7		0	1	1	1		
8		1	0	0	0		
9		1	0	0	1		
10		1	0	1	0		
11		1	0	1	1		
12		1	1	0	0		
13		1	1	0	1		
14		1	1	1	0		
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	Decimal			Bin	ary		
n	$n \mod 3$	n_3	n_2	n_1	n_0	f_1	f_0
0	0	0	0	0	0	0	0
1	1	0	0	0	1	0	1
3	2	0	0	1	0	1	0
3		0	0	1	1		
4		0	1	0	0		
5		0	1	0	1		
6		0	1	1	0		
8		0	1	1	1		
8		1	0	0	0		
9		1	0	0	1		
10		1	0	1	0		
11		1	0	1	1		
12		1	1	0	0		
13		1	1	0	1		
14		1	1	1	0		
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			Binary							
	Decimal			Bin	ary					
n	$n \mod 3$	n_3	n_2	n_1	n_0	f_1	f_0			
0	0	0	0	0	0	0	0			
1	1	0	0	0	1	0	1			
$\begin{vmatrix} 2\\ 3 \end{vmatrix}$	2	0	0	1	0	1	0			
3	0	0	0	1	1	0	0			
4		0	1	0	0					
5		0	1	0	1					
6		0	1	1	0					
7		0	1	1	1					
8		1	0	0	0					
9		1	0	0	1					
10		1	0	1	0					
11		1	0	1	1					
12		1	1	0	0					
13		1	1	0	1					
14		1	1	1	0					
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	Decimal			Bin	ary		
n	$n \mod 3$	n_3	n_2	n_1	n_0	f_1	f_0
0	0	0	0	0	0	0	0
1	1	0	0	0	1	0	1
3	2	0	0	1	0	1	0
3	0	0	0	1	1	0	0
4	1	0	1	0	0	0	1
5		0	1	0	1		
6		0	1	1	0		
7		0	1	1	1		
8		1	0	0	0		
9		1	0	0	1		
10		1	0	1	0		
11		1	0	1	1		
12		1	1	0	0		
13		1	1	0	1		
14		1	1	1	0		
15		1	1	1	1		

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		Binary							
	Decimal			Bin	ary				
n	$n \mod 3$	n_3	n_2	n_1	n_0	f_1	f_0		
0	0	0	0	0	0	0	0		
1	1	0	0	0	1	0	1		
3	2	0	0	1	0	1	0		
3	0	0	0	1	1	0	0		
4	1	0	1	0	0	0	1		
5	2	0	1	0	1	1	0		
6		0	1	1	0				
7		0	1	1	1				
8		1	0	0	0				
9		1	0	0	1				
10		1	0	1	0				
11		1	0	1	1				
12		1	1	0	0				
13		1	1	0	1				
14		1	1	1	0				
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	Decimal			Bin	ary		
n	$n \mod 3$	n_3	n_2	n_1	n_0	f_1	f_0
0	0	0	0	0	0	0	0
1	1	0	0	0	1	0	1
$\begin{vmatrix} 2\\ 3 \end{vmatrix}$	2	0	0	1	0	1	0
3	0	0	0	1	1	0	0
4	1	0	1	0	0	0	1
5	2	0	1	0	1	1	0
6	0	0	1	1	0	0	0
7		0	1	1	1		
8		1	0	0	0		
9		1	0	0	1		
10		1	0	1	0		
11		1	0	1	1		
12		1	1	0	0		
13		1	1	0	1		
14		1	1	1	0		
15		1	1	1	1		

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	Decimal			Bin	ary		
n	$n \mod 3$	n_3	n_2	n_1	n_0	f_1	f_0
0	0	0	0	0	0	0	0
1	1	0	0	0	1	0	1
2	2	0	0	1	0	1	0
3	0	0	0	1	1	0	0
4	1	0	1	0	0	0	1
5	2	0	1	0	1	1	0
6	0	0	1	1	0	0	0
7	1	0	1	1	1	0	1
8		1	0	0	0		
9		1	0	0	1		
10		1	0	1	0		
11		1	0	1	1		
12		1	1	0	0		
13		1	1	0	1		
14		1	1	1	0		
15		1	1	1	1		

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	Decimal			Bin	ary		
n	$n \mod 3$	n_3	n_2	n_1	n_0	f_1	f_0
0	0	0	0	0	0	0	0
1	1	0	0	0	1	0	1
2 3	2	0	0	1	0	1	0
3	0	0	0	1	1	0	0
4	1	0	1	0	0	0	1
5	2	0	1	0	1	1	0
6	0	0	1	1	0	0	0
7	1	0	1	1	1	0	1
8	2	1	0	0	0	1	0
9		1	0	0	1		
10		1	0	1	0		
11		1	0	1	1		
12		1	1	0	0		
13		1	1	0	1		
14		1	1	1	0		
15		1	1	1	1		

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[Decimal			Bin	ary		
n	$n \mod 3$	n_3	n_2	n_1	n_0	f_1	f_0
0	0	0	0	0	0	0	0
1	1	0	0	0	1	0	1
3	2	0	0	1	0	1	0
3	0	0	0	1	1	0	0
4	1	0	1	0	0	0	1
5	2	0	1	0	1	1	0
6	0	0	1	1	0	0	0
7	1	0	1	1	1	0	1
8	2	1	0	0	0	1	0
9	0	1	0	0	1	0	0
10		1	0	1	0		
11		1	0	1	1		
12		1	1	0	0		
13		1	1	0	1		
14		1	1	1	0		
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	Decimal			Bin	ary		
n	$n \mod 3$	n_3	n_2	n_1	n_0	f_1	f_0
0	0	0	0	0	0	0	0
1	1	0	0	0	1	0	1
2 3	2	0	0	1	0	1	0
3	0	0	0	1	1	0	0
4	1	0	1	0	0	0	1
5	2	0	1	0	1	1	0
6	0	0	1	1	0	0	0
8	1	0	1	1	1	0	1
	2	1	0	0	0	1	0
9	0	1	0	0	1	0	0
10	1	1	0	1	0	0	1
11		1	0	1	1		
12		1	1	0	0		
13		1	1	0	1		
14		1	1	1	0		
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		Decimal		Binary					
İ	n	$n \mod 3$	n_3	n_2	n_1	n_0	f_1	f_0	
ĺ	0	0	0	0	0	0	0	0	
1	1	1	0	0	0	1	0	1	
ĺ	2 3	2	0	0	1	0	1	0	
	3	0	0	0	1	1	0	0	
	4	1	0	1	0	0	0	1	
	5	2	0	1	0	1	1	0	
ſ	6	0	0	1	1	0	0	0	
	7	1	0	1	1	1	0	1	
ĺ	8	2	1	0	0	0	1	0	
	9	0	1	0	0	1	0	0	
ĺ	10	1	1	0	1	0	0	1	
	11	2	1	0	1	1	1	0	
ſ	12		1	1	0	0			
	13		1	1	0	1			
ĺ	14		1	1	1	0			
	15		1	1	1	1			

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		Decimal			Bin	ary		
	n	$n \mod 3$	n_3	n_2	n_1	n_0	f_1	f_0
ĺ	0	0	0	0	0	0	0	0
	1	1	0	0	0	1	0	1
	2	2	0	0	1	0	1	0
	3	0	0	0	1	1	0	0
	4	1	0	1	0	0	0	1
	5	2	0	1	0	1	1	0
	6	0	0	1	1	0	0	0
	7	1	0	1	1	1	0	1
ĺ	8	2	1	0	0	0	1	0
	9	0	1	0	0	1	0	0
ĺ	10	1	1	0	1	0	0	1
1	11	2	1	0	1	1	1	0
ĺ	12	0	1	1	0	0	0	0
	13		1	1	0	1		
	14		1	1	1	0		
	15		1	1	1	1		

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	Decimal		Binary					
n	$n \mod 3$	n_3	n_2	n_1	n_0	f_1	f_0	
0	0	0	0	0	0	0	0	
1	1	0	0	0	1	0	1	
2	2	0	0	1	0	1	0	
3	0	0	0	1	1	0	0	
4	1	0	1	0	0	0	1	
5	2	0	1	0	1	1	0	
6	0	0	1	1	0	0	0	
7	1	0	1	1	1	0	1	
8	2	1	0	0	0	1	0	
9	0	1	0	0	1	0	0	
10	1	1	0	1	0	0	1	
11	2	1	0	1	1	1	0	
12	0	1	1	0	0	0	0	
13	1	1	1	0	1	0	1	
14		1	1	1	0			
15		1	1	1	1			

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		Decimal		Binary					
Ī	n	$n \mod 3$	n_3	n_2	n_1	n_0	f_1	f_0	
Ī	0	0	0	0	0	0	0	0	
	1	1	0	0	0	1	0	1	
ſ	2	2	0	0	1	0	1	0	
	3	0	0	0	1	1	0	0	
ſ	4	1	0	1	0	0	0	1	
	5	2	0	1	0	1	1	0	
ſ	6	0	0	1	1	0	0	0	
	7	1	0	1	1	1	0	1	
ſ	8	2	1	0	0	0	1	0	
	9	0	1	0	0	1	0	0	
ſ	10	1	1	0	1	0	0	1	
	11	2	1	0	1	1	1	0	
	12	0	1	1	0	0	0	0	
	13	1	1	1	0	1	0	1	
ľ	14	2	1	1	1	0	1	0	
	15		1	1	1	1			

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	Decimal			Bin	ary		
n	$n \mod 3$	n_3	n_2	n_1	n_0	f_1	f_0
0	0	0	0	0	0	0	0
1	1	0	0	0	1	0	1
2	2	0	0	1	0	1	0
3	0	0	0	1	1	0	0
4	1	0	1	0	0	0	1
5	2	0	1	0	1	1	0
6	0	0	1	1	0	0	0
7	1	0	1	1	1	0	1
8	2	1	0	0	0	1	0
9	0	1	0	0	1	0	0
10	1	1	0	1	0	0	1
11	2	1	0	1	1	1	0
12	0	1	1	0	0	0	0
13	1	1	1	0	1	0	1
14	2	1	1	1	0	1	0
15	0	1	1	1	1	0	0

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	Decimal		Binary						
n	$n \mod 3$	n_3	n_2	n_1	n_0	f_1	f_0		
0	0	0	0	0	0	0	0		
1	1	0	0	0	1	0	1		
2	2	0	0	1	0	1	0		
3	0	0	0	1	1	0	0		
4	1	0	1	0	0	0	1		
5	2	0	1	0	1	1	0		
6	0	0	1	1	0	0	0		
7	1	0	1	1	1	0	1		
8	2	1	0	0	0	1	0		
9	0	1	0	0	1	0	0		
10	1	1	0	1	0	0	1		
11	2	1	0	1	1	1	0		
12	0	1	1	0	0	0	0		
13	1	1	1	0	1	0	1		
14	2	1	1	1	0	1	0		
15	0	1	1	1	1	0	0		

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mod 3 (truth table)

	Decimal		Binary					
n	$n \mod 3$	n_3	n_2	n_1	n_0	f_1	f_0	
0	0	0	0	0	0	0	0	
1	1	0	0	0	1	0	1	
2	2	0	0	1	0	1	0	
3	0	0	0	1	1	0	0	
4	1	0	1	0	0	0	1	
5	2	0	1	0	1	1	0	
6	0	0	1	1	0	0	0	
7	1	0	1	1	1	0	1	
8	2	1	0	0	0	1	0	
9	0	1	0	0	1	0	0	
10	1	1	0	1	0	0	1	
11	2	1	0	1	1	1	0	
12	0	1	1	0	0	0	0	
13	1	1	1	0	1	0	1	
14	2	1	1	1	0	1	0	
15	0	1	1	1	1		0	

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Stone's Theorem

Theorem

The boolean functions are definable.

Proof.

It is enough to show that the functions $f:\mathbb{B}^n\to\mathbb{B}$ are definable. We do this by induction.

 $\underline{n=0}$: A 0-ary function is a constant, that is either 0 or 1. Thus definability is immediate.

Proof.

$$\underline{n+1}$$
: Let $f: \mathbb{B}^{n+1} \to \mathbb{B}$ be a function. Let $f_0(x_{n-1},\ldots,x_0) = f(0,x_{n-1},\ldots,x_0)$ and $f_1(x_{n-1},\ldots,x_0) = f(1,x_{n-1},\ldots,x_0)$. By induction the functions f_0 and f_1 are definable, hence the function $f(x_n,\ldots,x_0) = (x_n + f_0(x_{n-1},\ldots,x_0)) \cdot (\bar{x}_n + f_1(x_{n-1},\ldots,x_0))$ is definable.

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mod 3 unwinding f_1 (step 1) extra

n_3	n_2	n_1	n_0	f_1
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	1
1	1	1	1	0

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mod 3 unwinding f_1 (step 2) extra

\bar{n}_3					
n_2	n_1	n_0	f_1^0		
0	0	0	0		
0	0	1	0		
0	1	0	1		
0	1	1	0		
1	0	0	0		
1	0	1	1		
1	1	0	0		
1	1	1	0		

	n	3		
n_2	n_1	n_0	f_1^1	
0	0	0	1	
0	0	1	0	
0	1	0	0	
0	1	1	1	
1	0	0	0	
1	0	1	0	
1	1	0	1	
1	1	1	0	

$$f_1 = \bar{n}_3 f_1^0 + n_3 f_1^1$$

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mod 3 unwinding f_1^0, f_1^1 (step 3) extra

	$\bar{n}_3\bar{n}_2$	
n_1	n_0	f_1^{00}
0	0	0
0	1	0
1	0	1
1	1	0

	$\bar{n}_3 n_2$	2	
n_1	n_0	f_1^{01}	
0	0	0	$f_1^0 = \bar{n}_3 \bar{n}_2 f_1^{00} + \bar{n}_3 \bar{n}_2 f_1^{02}$
0	1	1	$J_1 = n_3 n_2 J_1 + n_3 n_2 J_1$
1	0	0	
1	1	0	

	$n_3\bar{n}_2$	
n_1	n_0	f_1^{10}
0	0	1
0	1	0
1	0	0
1	1	1

$$f_1^0 = n_3 \bar{n}_2 f_1^{10} + n_3 \bar{n}_2 f_1^{11}$$

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\bar{n}_3	$\bar{n}_2 n_1$
n_0	f_1^{001}
0	1
1	0
f_1^{001}	$\bar{n} = \bar{n}_0$

$$\begin{array}{c|cccc}
\bar{n}_3 n_2 \bar{n}_1 \\
n_0 & f_1^{010} \\
0 & 0 \\
1 & 1 \\
f_1^{010} = n_0
\end{array}$$

$$\begin{array}{c|c}
\bar{n}_3 n_2 n_1 \\
n_0 & f_1^{011} \\
0 & 0 \\
1 & 0 \\
f_1^{011} = 0
\end{array}$$

$$\begin{array}{c|c} n_3 \bar{n}_2 \bar{n}_1 \\ \hline n_0 & f_1^{100} \\ \hline 0 & 1 \\ 1 & 0 \\ \hline \end{array}$$

$$\begin{array}{c|c}
n_3 \bar{n}_2 n_1 \\
n_0 & f_1^{101} \\
0 & 0 \\
1 & 1
\end{array}$$

$$\begin{array}{c|c}
n_3 n_2 \bar{n}_1 \\
\hline
 & f_1^{110} \\
0 & 0 \\
1 & 0 \\
\hline
 & f^{110} = 0
\end{array}$$

$$\begin{array}{c|c}
n_3 n_2 n_1 \\
\hline
n_0 & f_1^{111} \\
0 & 1 \\
1 & 0 \\
f_1^{111} = \bar{n}_0
\end{array}$$

$$f_1^{00} = \bar{n}_3 \bar{n}_2 \bar{n}_1 f_1^{000} + \bar{n}_3 \bar{n}_2 n_1 f_1^{001} \quad f_1^{01} = \bar{n}_3 n_2 \bar{n}_1 f_1^{010} + \bar{n}_3 n_2 n_1 f_1^{011}$$

$$f_1^{10} = n_3 \bar{n}_2 \bar{n}_1 f_1^{100} + n_3 \bar{n}_2 n_1 f_1^{101} \quad f_1^{11} = n_3 n_2 \bar{n}_1 f_1^{110} + n_3 n_2 n_1 f_1^{111}$$

- Note that we skipped the last unwinding
- We can do one variable, no need to go to zero variables
- But possible...

$$\begin{split} f_1 = & \bar{n}_3 \bar{n}_2 \bar{n}_1 f_1^{000} + \bar{n}_3 \bar{n}_2 n_1 f_1^{001} + \bar{n}_3 n_2 \bar{n}_1 f_1^{010} + \bar{n}_3 n_2 n_1 f_1^{011} + \\ & n_3 \bar{n}_2 \bar{n}_1 f_1^{100} + n_3 \bar{n}_2 n_1 f_1^{101} + n_3 n_2 \bar{n}_1 f_1^{110} + n_3 n_2 n_1 f_1^{111} \\ = & \bar{n}_3 \bar{n}_2 \bar{n}_1 \cdot 0 + \bar{n}_3 \bar{n}_2 n_1 \bar{n}_0 + \bar{n}_3 n_2 \bar{n}_1 n_0 + \bar{n}_3 n_2 n_1 \cdot 0 + \\ & n_3 \bar{n}_2 \bar{n}_1 \bar{n}_0 + n_3 \bar{n}_2 n_1 n_0 + n_3 n_2 \bar{n}_1 \cdot 0 + n_3 n_2 n_1 \bar{n}_0 \\ = & \bar{n}_3 \bar{n}_2 n_1 \bar{n}_0 + \bar{n}_3 n_2 \bar{n}_1 n_0 + \\ & n_3 \bar{n}_2 \bar{n}_1 \bar{n}_0 + n_3 \bar{n}_2 n_1 n_0 + n_3 n_2 n_1 \bar{n}_0 \end{split}$$

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n_3	$\overline{n_2}$	$\overline{n_1}$	$\overline{n_0}$	f_1
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	1
1	1	1	1	0

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n_3	n_2	$\overline{n_1}$	$\overline{n_0}$	f_1
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	1
1	1	1	1	0

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n_3	$\overline{n_2}$	$\overline{n_1}$	$\overline{n_0}$	f_1
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	1
1	1	1	1	0

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n_3	$\overline{n_2}$	$\overline{n_1}$	$\overline{n_0}$	f_1	m_2
0	0	0	0	0	0
0	0	0	1	0	0
0	0	1	0	1	1
0	0	1	1	0	0
0	1	0	0	0	0
0	1	0	1	1	0
0	1	1	0	0	0
0	1	1	1	0	0
1	0	0	0	1	0
1	0	0	1	0	0
1	0	1	0	0	0
1	0	1	1	1	0
1	1	0	0	0	0
1	1	0	1	0	0
1	1	1	0	1	0
1	1	1	1	0	0

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n	$\frac{1}{3}$ n	2	$\overline{n_1}$	$\overline{n_0}$	f_1	$ m_2 $
0) ()	0	0	0	0
0) ()	0	1	0	0
0) ()	1	0	1	1
0	0)	1	1	0	0
0) 1		0	0	0	0
0) 1		0	1	1	0
0) 1		1	0	0	0
0) 1		1	1	0	0
1	. 0)	0	0	1	0
1	. 0)	0	1	0	0
1	. ()	1	0	0	0
1	. ()	1	1	1	0
1	. 1		0	0	0	0
1	. 1		0	1	0	0
1	. 1		1	0	1	0
1	. 1		1	1	0	0

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n_3	$\overline{n_2}$	$\overline{n_1}$	$\overline{n_0}$	f_1	m_2
0	0	0	0	0	0
0	0	0	1	0	0
0	0	1	0	1	1
0	0	1	1	0	0
0	1	0	0	0	0
0	1	0	1	1	0
0	1	1	0	0	0
0	1	1	1	0	0
1	0	0	0	1	0
1	0	0	1	0	0
1	0	1	0	0	0
1	0	1	1	1	0
1	1	0	0	0	0
1	1	0	1	0	0
1	1	1	0	1	0
1	1	1	1	0	0

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n_3	n_2	$\overline{n_1}$	n_0	f_1	m_2	m_5
0	0	0	0	0	0	0
0	0	0	1	0	0	0
0	0	1	0	1	1	0
0	0	1	1	0	0	0
0	1	0	0	0	0	0
0	1	0	1	1	0	1
0	1	1	0	0	0	0
0	1	1	1	0	0	0
1	0	0	0	1	0	0
1	0	0	1	0	0	0
1	0	1	0	0	0	0
1	0	1	1	1	0	0
1	1	0	0	0	0	0
1	1	0	1	0	0	0
1	1	1	0	1	0	0
1	1	1	1	0	0	0

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n_3	n_2	n_1	n_0	f_1	m_2	m_5	
0	0	0	0	0	0	0	
0	0	0	1	0	0	0	
0	0	1	0	1	1	0	
0	0	1	1	0	0	0	
0	1	0	0	0	0	0	
0	1	0	1	1	0	1	
0	1	1	0	0	0	0	
0	1	1	1	0	0	0	
1	0	0	0	1	0	0	
1	0	0	1	0	0	0	
1	0	1	0	0	0	0	
1	0	1	1	1	0	0	
1	1	0	0	0	0	0	
1	1	0	1	0	0	0	
1	1	1	0	1	0	0	
1	1	1	1	0	0	0	

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n_3	n_2	$\overline{n_1}$	$\overline{n_0}$	f_1	m_2	m_5
0	0	0	0	0	0	0
0	0	0	1	0	0	0
0	0	1	0	1	1	0
0	0	1	1	0	0	0
0	1	0	0	0	0	0
0	1	0	1	1	0	1
0	1	1	0	0	0	0
0	1	1	1	0	0	0
1	0	0	0	1	0	0
1	0	0	1	0	0	0
1	0	1	0	0	0	0
1	0	1	1	1	0	0
1	1	0	0	0	0	0
1	1	0	1	0	0	0
1	1	1	0	1	0	0
1	1	1	1	0	0	0

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$\overline{n_3}$	$\overline{n_2}$	$\overline{n_1}$	$\overline{n_0}$	f_1	m_2	$\overline{m_5}$	m_8
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0
0	0	1	0	1	1	0	0
0	0	1	1	0	0	0	0
0	1	0	0	0	0	0	0
0	1	0	1	1	0	1	0
0	1	1	0	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	0	1	0	0	1
1	0	0	1	0	0	0	0
1	0	1	0	0	0	0	0
1	0	1	1	1	0	0	0
1	1	0	0	0	0	0	0
1	1	0	1	0	0	0	0
1	1	1	0	1	0	0	0
1	1	1	1	0	0	0	0

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n_3	n_2	n_1	$\overline{n_0}$	f_1	m_2	m_5	m_8
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0
0	0	1	0	1	1	0	0
0	0	1	1	0	0	0	0
0	1	0	0	0	0	0	0
0	1	0	1	1	0	1	0
0	1	1	0	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	0	1	0	0	1
1	0	0	1	0	0	0	0
1	0	1	0	0	0	0	0
1	0	1	1	1	0	0	0
1	1	0	0	0	0	0	0
1	1	0	1	0	0	0	0
1	1	1	0	1	0	0	0
1	1	1	1	0	0	0	0

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n_3	n_2	n_1	n_0	f_1	m_2	m_5	m_8	
0	0	0	0	0	0	0	0	
0	0	0	1	0	0	0	0	
0	0	1	0	1	1	0	0	
0	0	1	1	0	0	0	0	
0	1	0	0	0	0	0	0	
0	1	0	1	1	0	1	0	
0	1	1	0	0	0	0	0	
0	1	1	1	0	0	0	0	
1	0	0	0	1	0	0	1	
1	0	0	1	0	0	0	0	
1	0	1	0	0	0	0	0	
1	0	1	1	1	0	0	0	
1	1	0	0	0	0	0	0	
1	1	0	1	0	0	0	0	
1	1	1	0	1	0	0	0	
1	1	1	1	0	0	0	0	

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n_3	n_2	n_1	n_0	f_1	m_2	m_5	m_8	m_{11}	
0	0	0	0	0	0	0	0	0	
0	0	0	1	0	0	0	0	0	
0	0	1	0	1	1	0	0	0	
0	0	1	1	0	0	0	0	0	
0	1	0	0	0	0	0	0	0	
0	1	0	1	1	0	1	0	0	
0	1	1	0	0	0	0	0	0	
0	1	1	1	0	0	0	0	0	
1	0	0	0	1	0	0	1	0	
1	0	0	1	0	0	0	0	0	
1	0	1	0	0	0	0	0	0	
1	0	1	1	1	0	0	0	1	
1	1	0	0	0	0	0	0	0	
1	1	0	1	0	0	0	0	0	
1	1	1	0	1	0	0	0	0	
1	1	1	1	0	0	0	0	0	

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n_3	n_2	n_1	n_0	f_1	m_2	m_5	m_8	m_{11}
0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0
0	0	1	0	1	1	0	0	0
0	0	1	1	0	0	0	0	0
0	1	0	0	0	0	0	0	0
0	1	0	1	1	0	1	0	0
0	1	1	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0
1	0	0	0	1	0	0	1	0
1	0	0	1	0	0	0	0	0
1	0	1	0	0	0	0	0	0
1	0	1	1	1	0	0	0	1
1	1	0	0	0	0	0	0	0
1	1	0	1	0	0	0	0	0
1	1	1	0	1	0	0	0	0
1	1	1	1	0	0	0	0	0

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n_3	n_2	n_1	n_0	f_1	m_2	m_5	m_8	m_{11}	
0	0	0	0	0	0	0	0	0	
0	0	0	1	0	0	0	0	0	
0	0	1	0	1	1	0	0	0	
0	0	1	1	0	0	0	0	0	
0	1	0	0	0	0	0	0	0	
0	1	0	1	1	0	1	0	0	
0	1	1	0	0	0	0	0	0	
0	1	1	1	0	0	0	0	0	
1	0	0	0	1	0	0	1	0	
1	0	0	1	0	0	0	0	0	
1	0	1	0	0	0	0	0	0	
1	0	1	1	1	0	0	0	1	
1	1	0	0	0	0	0	0	0	
1	1	0	1	0	0	0	0	0	
1	1	1	0	1	0	0	0	0	
1	1	1	1	0	0	0	0	0	

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n_3	n_2	n_1	n_0	$ f_1 $	m_2	m_5	m_8	m_{11}	m_{14}
0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0
0	0	1	0	1	1	0	0	0	0
0	0	1	1	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0
0	1	0	1	1	0	1	0	0	0
0	1	1	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0
1	0	0	0	1	0	0	1	0	0
1	0	0	1	0	0	0	0	0	0
1	0	1	0	0	0	0	0	0	0
1	0	1	1	1	0	0	0	1	0
1	1	0	0	0	0	0	0	0	0
1	1	0	1	0	0	0	0	0	0
1	1	1	0	1	0	0	0	0	1
1	1	1	1	0	0	0	0	0	0

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	$\overline{n_3}$	$\overline{n_2}$	$\overline{n_1}$	$\overline{n_0}$	f_1	$ m_2 $	m_5	m_8	m_{11}	m_{14}
ĺ	0	0	0	0	0	0	0	0	0	0
	0	0	0	1	0	0	0	0	0	0
ĺ	0	0	1	0	1	1	0	0	0	0
	0	0	1	1	0	0	0	0	0	0
ĺ	0	1	0	0	0	0	0	0	0	0
	0	1	0	1	1	0	1	0	0	0
	0	1	1	0	0	0	0	0	0	0
	0	1	1	1	0	0	0	0	0	0
	1	0	0	0	1	0	0	1	0	0
1	1	0	0	1	0	0	0	0	0	0
	1	0	1	0	0	0	0	0	0	0
1	1	0	1	1	1	0	0	0	1	0
	1	1	0	0	0	0	0	0	0	0
	1	1	0	1	0	0	0	0	0	0
	1	1	1	0	1	0	0	0	0	1
	1	1	1	1	0	0	0	0	0	0

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ield of Sets

n_3	n_2	n_1	n_0	f_1	m_2	m_5	m_8	m_{11}	m_{14}
0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0
0	0	1	0	1	1	0	0	0	0
0	0	1	1	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0
0	1	0	1	1	0	1	0	0	0
0	1	1	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0
1	0	0	0	1	0	0	1	0	0
1	0	0	1	0	0	0	0	0	0
1	0	1	0	0	0	0	0	0	0
1	0	1	1	1	0	0	0	1	0
1	1	0	0	0	0	0	0	0	0
1	1	0	1	0	0	0	0	0	0
1	1	1	0	1	0	0	0	0	1
1	1	1	1	0	0	0	0	0	0

$$f_1(n_3, n_2, n_1, n_0) = \sum_{\bar{n}_3\bar{n}_2 n_1\bar{n}_0 + \bar{n}_3 n_2\bar{n}_1 \bar{n}_0 + n_3\bar{n}_2\bar{n}_1\bar{n}_0 + n_3\bar{n}_2n_1\bar{n}_0 + n_3\bar{n}_2n_1n_0 + n_3n_2n_1\bar{n}_0}$$

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_				
n_3	n_2	n_1	n_0	
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1			0	1
	0	0		
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	1
1	1	1	1	0

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Field of Sets

_				
n_3	n_2	n_1	n_0	$ f_1 $
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	1
1	1	1	1	0
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Field of Sets

n_3	n_2	n_1	n_0	f_1
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	1
1 -				
1	1	1	1	0

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Field of Sets

_					
n_3	n_2	n_1	n_0	f_1	M_0
0	0	0	0	0	0
0	0	0	1	0	1
0	0	1	0	1	1
0	0	1	1	0	1
0	1	0	0	0	1
0	1	0	1	1	1
0	1	1	0	0	1
0	1	1	1	0	1
1				1	1
_	0	0	0		
1	0	0	1	0	1
1	0	1	0	0	1
1	0	1	1	1	1
1	1	0	0	0	1
1	1	0	1	0	1
1	1	1	0	1	1
1	1	1	1	0	1

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n_3	n_2	n_1	n_0	f_1	M_0
0	0	0	0	0	0
0	0	0	1	0	1
0	0	1	0	1	1
0	0	1	1	0	1
0	1	0	0	0	1
0	1	0	1	1	1
0	1	1	0	0	1
0	1	1	1	0	1
1	0	0	0	1	1
1	0	0	1	0	1
1	0	1	0	0	1
1	0	1	1	1	1
1	1	0	0	0	1
1	1	0	1	0	1
1	1	1	0	1	1
1	1	1	1	0	1

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n_3	n_2	n_1	n_0	f_1	M_0
0	0	0	0	0	0
0	0	0	1	0	1
0	0	1	0	1	1
0	0	1	1	0	1
0	1	0	0	0	1
0	1	0	1	1	1
0	1	1	0	0	1
0	1	1	1	0	1
1	0	0	0	1	1
1	0	0	1	0	1
1	0	1	0	0	1
1	0	1	1	1	1
1	1	0	0	0	1
1	1	0	1	0	1
1	1	1	0	1	1
1	1	1	1	0	1

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Field of Sets

n_3	n_2	n_1	n_0	f_1	M_0	M_1
0	0	0	0	0	0	1
0	0	0	1	0	1	0
0	0	1	0	1	1	1
0	0	1	1	0	1	1
0	1	0	0	0	1	1
0	1	0	1	1	1	1
0	1	1	0	0	1	1
0	1	1	1	0	1	1
1	0	0	0	1	1	1
1	0	0	1	0	1	1
1	0	1	0	0	1	1
1	0	1	1	1	1	1
1	1	0	0	0	1	1
1	1	0	1	0	1	1
1	1	1	0	1	1	1
1	1	1	1	0	1	1

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				ſ	1/	1/
_				-	M_0	
0	0	0	0	0	0	1
0	0	0	1	0	1	0
0	0	1	0	1	1	1
0	0	1	1	0	1	1
0	1	0	0	0	1	1
0	1	0	1	1	1	1
0	1	1	0	0	1	1
0	1	1	1	0	1	1
1	0	0	0	1	1	1
1	0	0	1	0	1	1
1	0	1	0	0	1	1
_	~	_				-
1	0	1	1	1	1	1
1	1	0	0	0	1	1
1	1	0	1	0	1	1
1	1	1	0	1	1	1
1	1	1	1	0	1	1

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Field of Sets

					_	
n_3	n_2	n_1	n_0	$ f_1 $	M_0	M_1
0	0	0	0	0	0	1
0	0	0	1	0	1	0
0	0	1	0	1	1	1
0	0	1	1	0	1	1
0	1	0	0	0	1	1
0	1	0	1	1	1	1
0	1	1	0	0	1	1
0	1	1	1	0	1	1
1	0	0	0	1	1	1
						_
1	0	0	1	0	1	1
1	0	1	0	0	1	1
1	0	1	1	1	1	1
1	1	0	0	0	1	1
1	1	0	1	0	1	1
1	1	1	0	1	1	1
1	1	1	1	0	1	1

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n_3	n_2	n_1	n_0	f_1	M_0	M_1	M_3
0	0	0	0	0	0	1	1
0	0	0	1	0	1	0	1
0	0	1	0	1	1	1	1
0	0	1	1	0	1	1	0
0	1	0	0	0	1	1	1
0	1	0	1	1	1	1	1
0	1	1	0	0	1	1	1
0	1	1	1	0	1	1	1
1	0	0	0	1	1	1	1
1	0	0	1	0	1	1	1
1	0	1	0	0	1	1	1
1	0	1	1	1	1	1	1
1	1	0	0	0	1	1	1
1	1	0	1	0	1	1	1
1	1	1	0	1	1	1	1
1	1	1	1	0	1	1	1

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Field of Sets

_							
n_3	n_2	n_1	n_0	f_1	M_0	M_1	M_3
0	0	0	0	0	0	1	1
0	0	0	1	0	1	0	1
0	0	1	0	1	1	1	1
0	0	1	1	0	1	1	0
0	1	0	0	0	1	1	1
0	1	0	1	1	1	1	1
0	1	1	0	0	1	1	1
0	1	1	1	0	1	1	1
1	0	0	0	1	1	1	1
_						_	-
1	0	0	1	0	1	1	1
1	0	1	0	0	1	1	1
1	0	1	1	1	1	1	1
1	1	0	0	0	1	1	1
1	1	0	1	0	1	1	1
1	1	1	0	1	1	1	1
1	1	1	1	0	1	1	1

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Field of Sets

				-	3.6	3.6	1.6
n_3	n_2	n_1	n_0	f_1	M_0	M_1	M_3
0	0	0	0	0	0	1	1
0	0	0	1	0	1	0	1
0	0	1	0	1	1	1	1
0	0	1	1	0	1	1	0
0	1	0	0	0	1	1	1
0	1	0	1	1	1	1	1
0	1	1	0	0	1	1	1
0	1	1	1	0	1	1	1
1	0	0	0	1	1	1	1
1	0	0	1	0	1	1	1
1	0	1	0	0	1	1	1
1	0	1	1	1	1	1	1
1	1	0	0	0	1	1	1
1	1	0	1	0	1	1	1
1	1	1	0	1	1	1	1
1	1	1	1	0	1	1	1

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Field of Sets

				£	11	11	11	11
n_3	n_2	n_1	n_0	J_1	M_0		M_3	
0	0	0	0	0	0	1	1	1
0	0	0	1	0	1	0	1	1
0	0	1	0	1	1	1	1	1
0	0	1	1	0	1	1	0	1
0	1	0	0	0	1	1	1	0
0	1	0	1	1	1	1	1	1
0	1	1	0	0	1	1	1	1
0	1	1	1	0	1	1	1	1
1 "							_	
1	0	0	0	1	1	1	1	1
1	0	0	1	0	1	1	1	1
1	0	1	0	0	1	1	1	1
1	0	1	1	1	1	1	1	1
1	1	0	0	0	1	1	1	1
1	1	0	1	0	1	1	1	1
1	1	1	0	1	1	1	1	1
1	1	1	1	0	1	1	1	1

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Field of Sets

n_3	n_2	n_1	n_0	f_1	M_0	M_1	M_3	M_4
0	0	0	0	0	0	1	1	1
0	0	0	1	0	1	0	1	1
0	0	1	0	1	1	1	1	1
0	0	1	1	0	1	1	0	1
0	1	0	0	0	1	1	1	0
0	1	0	1	1	1	1	1	1
0	1	1	0	0	1	1	1	1
0	1	1	1	0	1	1	1	1
1	0	0	0	1	1	1	1	1
1	0	0	1	0	1	1	1	1
1	0	1	0	0	1	1	1	1
1	0	1	1	1	1	1	1	1
1	1	0	0	0	1	1	1	1
1	1	0	1	0	1	1	1	1
1	1	1	0	1	1	1	1	1
1	1	1	1	0	1	1	1	1

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Field of Sets

n_3	n_2	n_1	n_0	f_1	M_0	$\overline{M_1}$	M_3	$\overline{M_4}$
0	0	0	0	0	0	1	1	1
0	0	0	1	0	1	0	1	1
0	0	1	0	1	1	1	1	1
0	0	1	1	0	1	1	0	1
0	1	0	0	0	1	1	1	0
0	1	0	1	1	1	1	1	1
0	1	1	0	0	1	1	1	1
0	1	1	1	0	1	1	1	1
1	0	0	0	1	1	1	1	1
1	0	0	1	0	1	1	1	1
1	0	1	0	0	1	1	1	1
1	0	1	1	1	1	1	1	1
1	1	0	0	0	1	1	1	1
1	1	0	1	0	1	1	1	1
1	1	1	0	1	1	1	1	1
1	1	1	1	0	1	1	1	1

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Field of Sets

_									
n_3	n_2	n_1	n_0	f_1	M_0	M_1	M_3	M_4	M_6
0	0	0	0	0	0	1	1	1	1
0	0	0	1	0	1	0	1	1	1
0	0	1	0	1	1	1	1	1	1
0	0	1	1	0	1	1	0	1	1
0	1	0	0	0	1	1	1	0	1
0	1	0	1	1	1	1	1	1	1
0	1	1	0	0	1	1	1	1	0
0	1	1	1	0	1	1	1	1	1
1	0	0	0	1	1	1	1	1	1
1	0	0	1	0	1	1	1	1	1
1	0	1	0	0	1	1	1	1	1
1	0	1	1	1	1	1	1	1	1
1	1	0	0	0	1	1	1	1	1
1	1	0	1	0	1	1	1	1	1
1	1	1	0	1	1	1	1	1	1
1	1	1	1	0	1	1	1	1	1

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_									
n_3	n_2	n_1	n_0	f_1	M_0	M_1	M_3	M_4	M_6
0	0	0	0	0	0	1	1	1	1
0	0	0	1	0	1	0	1	1	1
0	0	1	0	1	1	1	1	1	1
0	0	1	1	0	1	1	0	1	1
0	1	0	0	0	1	1	1	0	1
0	1	0	1	1	1	1	1	1	1
0	1	1	0	0	1	1	1	1	0
0	1	1	1	0	1	1	1	1	1
1	0	0	0	1	1	1	1	1	1
1	0	0	1	0	1	1	1	1	1
1	0	1	0	0	1	1	1	1	1
1	0	1	1	1	1	1	1	1	1
1	1	0	0	0	1	1	1	1	1
1	1	0	1	0	1	1	1	1	1
1	1	1	0	1	1	1	1	1	1
1	1	1	1	0	1	1	1	1	1

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Field of Sets

_									
n_3	n_2	n_1	n_0	f_1	M_0	M_1	M_3	M_4	M_6
0	0	0	0	0	0	1	1	1	1
0	0	0	1	0	1	0	1	1	1
0	0	1	0	1	1	1	1	1	1
0	0	1	1	0	1	1	0	1	1
0	1	0	0	0	1	1	1	0	1
0	1	0	1	1	1	1	1	1	1
0	1	1	0	0	1	1	1	1	0
0	1	1	1	0	1	1	1	1	1
1	0	0	0	1	1	1	1	1	1
1	0	0	1	0	1	1	1	1	1
1	0	1	0	0	1	1	1	1	1
1	0	1	1	1	1	1	1	1	1
1	1	0	0	0	1	1	1	1	1
1	1	0	1	0	1	1	1	1	1
1	1	1	0	1	1	1	1	1	1
1	1	1	1	0	1	1	1	1	1

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n_3	n_2	n_1	n_0	f_1	M_0	M_1	M_3	M_4	M_6	M_7
0	0	0	0	0	0	1	1	1	1	1
0	0	0	1	0	1	0	1	1	1	1
0	0	1	0	1	1	1	1	1	1	1
0	0	1	1	0	1	1	0	1	1	1
0	1	0	0	0	1	1	1	0	1	1
0	1	0	1	1	1	1	1	1	1	1
0	1	1	0	0	1	1	1	1	0	1
0	1	1	1	0	1	1	1	1	1	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	0	1	1	1	1	1	1
1	0	1	0	0	1	1	1	1	1	1
1	0	1	1	1	1	1	1	1	1	1
1	1	0	0	0	1	1	1	1	1	1
1	1	0	1	0	1	1	1	1	1	1
1	1	1	0	1	1	1	1	1	1	1
1	1	1	1	0	1	1	1	1	1	1

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Field of Sets

_				_							
n_3	n_2	n_1	n_0	$ f_1 $	M_0	M_1	M_3	M_4	M_6	M_7	
0	0	0	0	0	0	1	1	1	1	1	
0	0	0	1	0	1	0	1	1	1	1	
0	0	1	0	1	1	1	1	1	1	1	
0	0	1	1	0	1	1	0	1	1	1	
0	1	0	0	0	1	1	1	0	1	1	
0	1	0	1	1	1	1	1	1	1	1	
0	1	1	0	0	1	1	1	1	0	1	
0	1	1	1	0	1	1	1	1	1	0	
1	0	0	0	1	1	1	1	1	1	1	
1	0	0	1	0	1	1	1	1	1	1	
1	0	1	0	0	1	1	1	1	1	1	
1	0	1	1	1	1	1	1	1	1	1	
1	1	0	0	0	1	1	1	1	1	1	
1	1	0	1	0	1	1	1	1	1	1	
1	1	1	0	1	1	1	1	1	1	1	
1	1	1	1	0	1	1	1	1	1	1	

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n-Bits Binary Adder

Field of Sets

$\mod 3$ f_1 in canonical POS

_										
n_3	n_2	n_1	n_0	$ f_1 $	M_0	M_1	M_3	M_4	M_6	M_7
0	0	0	0	0	0	1	1	1	1	1
0	0	0	1	0	1	0	1	1	1	1
0	0	1	0	1	1	1	1	1	1	1
0	0	1	1	0	1	1	0	1	1	1
0	1	0	0	0	1	1	1	0	1	1
0	1	0	1	1	1	1	1	1	1	1
0	1	1	0	0	1	1	1	1	0	1
0	1	1	1	0	1	1	1	1	1	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	0	1	1	1	1	1	1
1	0	1	0	0	1	1	1	1	1	1
1	0	1	1	1	1	1	1	1	1	1
1	1	0	0	0	1	1	1	1	1	1
1	1	0	1	0	1	1	1	1	1	1
1	1	1	0	1	1	1	1	1	1	1
1	1	1	1	0	1	1	1	1	1	1

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Boolean Algebras ©C.M.

$mod 3 f_1$ in canonical POS

_												
n_3	n_2	n_1	n_0	f_1	M_0	M_1	M_3	M_4	M_6	M_7	M_9	
0	0	0	0	0	0	1	1	1	1	1	1	
0	0	0	1	0	1	0	1	1	1	1	1	
0	0	1	0	1	1	1	1	1	1	1	1	
0	0	1	1	0	1	1	0	1	1	1	1	
0	1	0	0	0	1	1	1	0	1	1	1	
0	1	0	1	1	1	1	1	1	1	1	1	
0	1	1	0	0	1	1	1	1	0	1	1	
0	1	1	1	0	1	1	1	1	1	0	1	
1	0	0	0	1	1	1	1	1	1	1	1	
1	0	0	1	0	1	1	1	1	1	1	0	
1	0	1	0	0	1	1	1	1	1	1	1	
1	0	1	1	1	1	1	1	1	1	1	1	
1	1	0	0	0	1	1	1	1	1	1	1	
1	1	0	1	0	1	1	1	1	1	1	1	
1	1	1	0	1	1	1	1	1	1	1	1	
1	1	1	1	0	1	1	1	1	1	1	1	

Unwinding

Adder

Boolean Algebras ©C.M.

mod $3 f_1$ in canonical POS

n_3	n_2	n_1	n_0	f_1	M_0	M_1	M_3	M_4	M_6	M_7	M_9
0	0	0	0	0	0	1	1	1	1	1	1
0	0	0	1	0	1	0	1	1	1	1	1
0	0	1	0	1	1	1	1	1	1	1	1
0	0	1	1	0	1	1	0	1	1	1	1
0	1	0	0	0	1	1	1	0	1	1	1
0	1	0	1	1	1	1	1	1	1	1	1
0	1	1	0	0	1	1	1	1	0	1	1
0	1	1	1	0	1	1	1	1	1	0	1
1	0	0	0	1	1	1	1	1	1	1	1
1	0	0	1	0	1	1	1	1	1	1	0
1	0	1	0	0	1	1	1	1	1	1	1
1	0	1	1	1	1	1	1	1	1	1	1
1	1	0	0	0	1	1	1	1	1	1	1
1	1	0	1	0	1	1	1	1	1	1	1
1	1	1	0	1	1	1	1	1	1	1	1
1	1	1	1	0	1	1	1	1	1	1	1

Unwinding

Adder

$mod 3 f_1 in canonical POS$

_				_								
n_3	n_2	n_1	n_0	$ f_1 $	M_0	M_1	M_3	M_4	M_6	M_7	M_9	
0	0	0	0	0	0	1	1	1	1	1	1	
0	0	0	1	0	1	0	1	1	1	1	1	
0	0	1	0	1	1	1	1	1	1	1	1	
0	0	1	1	0	1	1	0	1	1	1	1	
0	1	0	0	0	1	1	1	0	1	1	1	
0	1	0	1	1	1	1	1	1	1	1	1	
0	1	1	0	0	1	1	1	1	0	1	1	
0	1	1	1	0	1	1	1	1	1	0	1	
1	0	0	0	1	1	1	1	1	1	1	1	
1	0	0	1	0	1	1	1	1	1	1	0	
1	0	1	0	0	1	1	1	1	1	1	1	
1	0	1	1	1	1	1	1	1	1	1	1	
1	1	0	0	0	1	1	1	1	1	1	1	
1	1	0	1	0	1	1	1	1	1	1	1	
1	1	1	0	1	1	1	1	1	1	1	1	
1	1	1	1	0	1	1	1	1	1	1	1	

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Ainimal BA

Definable function

Seven segment

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n-Bits Binary Adder

Field of Sets

$\mod 3 \ f_1$ in canonical POS

n_3	n_2	n_1	n_0	f_1	M_0	M_1	M_3	M_4	M_6	M_7	M_9	M_{10}
0	0	0	0	0	0	1	1	1	1	1	1	1
0	0	0	1	0	1	0	1	1	1	1	1	1
0	0	1	0	1	1	1	1	1	1	1	1	1
0	0	1	1	0	1	1	0	1	1	1	1	1
0	1	0	0	0	1	1	1	0	1	1	1	1
0	1	0	1	1	1	1	1	1	1	1	1	1
0	1	1	0	0	1	1	1	1	0	1	1	1
0	1	1	1	0	1	1	1	1	1	0	1	1
1	0	0	0	1	1	1	1	1	1	1	1	1
1	0	0	1	0	1	1	1	1	1	1	0	1
1	0	1	0	0	1	1	1	1	1	1	1	0
1	0	1	1	1	1	1	1	1	1	1	1	1
1	1	0	0	0	1	1	1	1	1	1	1	1
1	1	0	1	0	1	1	1	1	1	1	1	1
1	1	1	0	1	1	1	1	1	1	1	1	1
1	1	1	1	0	1	1	1	1	1	1	1	1

Boolean Algebra

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Field of Sets

mod $3 f_1$ in canonical POS

n_3	n_2	n_1	n_0	f_1	M_0	M_1	M_3	M_4	M_6	M_7	M_9	M_{10}	
0	0	0	0	0	0	1	1	1	1	1	1	1	
0	0	0	1	0	1	0	1	1	1	1	1	1	
0	0	1	0	1	1	1	1	1	1	1	1	1	
0	0	1	1	0	1	1	0	1	1	1	1	1	
0	1	0	0	0	1	1	1	0	1	1	1	1	
0	1	0	1	1	1	1	1	1	1	1	1	1	
0	1	1	0	0	1	1	1	1	0	1	1	1	
0	1	1	1	0	1	1	1	1	1	0	1	1	
1	0	0	0	1	1	1	1	1	1	1	1	1	
1	0	0	1	0	1	1	1	1	1	1	0	1	
1	0	1	0	0	1	1	1	1	1	1	1	0	
1	0	1	1	1	1	1	1	1	1	1	1	1	
1	1	0	0	0	1	1	1	1	1	1	1	1	
1	1	0	1	0	1	1	1	1	1	1	1	1	
1	1	1	0	1	1	1	1	1	1	1	1	1	
1	1	1	1	0	1	1	1	1	1	1	1	1	

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$mod 3 f_1 in canonical POS$

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n_3	n_2	n_1	n_0	$ f_1 $	$ M_0 $	M_1	M_3	M_4	M_6	M_7	M_9	M_{10}	
0	0	0	0	0	0	1	1	1	1	1	1	1	
0	0	0	1	0	1	0	1	1	1	1	1	1	
0	0	1	0	1	1	1	1	1	1	1	1	1	
0	0	1	1	0	1	1	0	1	1	1	1	1	
0	1	0	0	0	1	1	1	0	1	1	1	1	
0	1	0	1	1	1	1	1	1	1	1	1	1	
0	1	1	0	0	1	1	1	1	0	1	1	1	
0	1	1	1	0	1	1	1	1	1	0	1	1	
1	0	0	0	1	1	1	1	1	1	1	1	1	
1	0	0	1	0	1	1	1	1	1	1	0	1	
1	0	1	0	0	1	1	1	1	1	1	1	0	
1	0	1	1	1	1	1	1	1	1	1	1	1	
1	1	0	0	0	1	1	1	1	1	1	1	1	
1	1	0	1	0	1	1	1	1	1	1	1	1	
1	1	1	0	1	1	1	1	1	1	1	1	1	
1	1	1	1	0	1	1	1	1	1	1	1	1	

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Field of Sets

mod $3 f_1$ in canonical POS

n_3	n_2	n_1	n_0	f_1	M_0	M_1	M_3	M_4	M_6	M_7	M_9	M_{10}	M_{12}
0	0	0	0	0	0	1	1	1	1	1	1	1	1
0	0	0	1	0	1	0	1	1	1	1	1	1	1
0	0	1	0	1	1	1	1	1	1	1	1	1	1
0	0	1	1	0	1	1	0	1	1	1	1	1	1
0	1	0	0	0	1	1	1	0	1	1	1	1	1
0	1	0	1	1	1	1	1	1	1	1	1	1	1
0	1	1	0	0	1	1	1	1	0	1	1	1	1
0	1	1	1	0	1	1	1	1	1	0	1	1	1
1	0	0	0	1	1	1	1	1	1	1	1	1	1
1	0	0	1	0	1	1	1	1	1	1	0	1	1
1	0	1	0	0	1	1	1	1	1	1	1	0	1
1	0	1	1	1	1	1	1	1	1	1	1	1	1
1	1	0	0	0	1	1	1	1	1	1	1	1	0
1	1	0	1	0	1	1	1	1	1	1	1	1	1
1	1	1	0	1	1	1	1	1	1	1	1	1	1
1	1	1	1	0	1	1	1	1	1	1	1	1	1

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Truth tables

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 $n ext{-Bits Binary}$ Adder

Field of Sets

Boolean Algebras ©C.M.

mod $3 f_1$ in canonical POS

n_3	n_2	n_1	n_0	f_1	M_0	M_1	M_3	M_4	M_6	M_7	M_9	M_{10}	M_{12}	
0	0	0	0	0	0	1	1	1	1	1	1	1	1	
0	0	0	1	0	1	0	1	1	1	1	1	1	1	
0	0	1	0	1	1	1	1	1	1	1	1	1	1	
0	0	1	1	0	1	1	0	1	1	1	1	1	1	
0	1	0	0	0	1	1	1	0	1	1	1	1	1	
0	1	0	1	1	1	1	1	1	1	1	1	1	1	
0	1	1	0	0	1	1	1	1	0	1	1	1	1	
0	1	1	1	0	1	1	1	1	1	0	1	1	1	
1	0	0	0	1	1	1	1	1	1	1	1	1	1	
1	0	0	1	0	1	1	1	1	1	1	0	1	1	
1	0	1	0	0	1	1	1	1	1	1	1	0	1	
1	0	1	1	1	1	1	1	1	1	1	1	1	1	
1	1	0	0	0	1	1	1	1	1	1	1	1	0	
1	1	0	1	0	1	1	1	1	1	1	1	1	1	
1	1	1	0	1	1	1	1	1	1	1	1	1	1	
1	1	1	1	0	1	1	1	1	1	1	1	1	1	

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$mod 3 f_1$ in canonical POS

n_3	n_2	n_1	n_0	f_1	M_0	M_1	M_3	M_4	M_6	M_7	M_9	M_{10}	M_{12}	
0	0	0	0	0	0	1	1	1	1	1	1	1	1	
0	0	0	1	0	1	0	1	1	1	1	1	1	1	
0	0	1	0	1	1	1	1	1	1	1	1	1	1	
0	0	1	1	0	1	1	0	1	1	1	1	1	1	
0	1	0	0	0	1	1	1	0	1	1	1	1	1	
0	1	0	1	1	1	1	1	1	1	1	1	1	1	
0	1	1	0	0	1	1	1	1	0	1	1	1	1	
0	1	1	1	0	1	1	1	1	1	0	1	1	1	
1	0	0	0	1	1	1	1	1	1	1	1	1	1	
1	0	0	1	0	1	1	1	1	1	1	0	1	1	
1	0	1	0	0	1	1	1	1	1	1	1	0	1	
1	0	1	1	1	1	1	1	1	1	1	1	1	1	
1	1	0	0	0	1	1	1	1	1	1	1	1	0	
1	1	0	1	0	1	1	1	1	1	1	1	1	1	
1	1	1	0	1	1	1	1	1	1	1	1	1	1	
1	1	1	1	0	1	1	1	1	1	1	1	1	1	

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Field of Sets

Stone's Theorem

$mod 3 f_1 in canonical POS$

n_3	n_2	n_1	n_0	f_1	M_0	M_1	M_3	M_4	M_6	M_7	M_9	M_{10}	M_{12}	M_{13}	
0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	
0	0	0	1	0	1	0	1	1	1	1	1	1	1	1	
0	0	1	0	1	1	1	1	1	1	1	1	1	1	1	
0	0	1	1	0	1	1	0	1	1	1	1	1	1	1	
0	1	0	0	0	1	1	1	0	1	1	1	1	1	1	
0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	
0	1	1	0	0	1	1	1	1	0	1	1	1	1	1	
0	1	1	1	0	1	1	1	1	1	0	1	1	1	1	
1	0	0	0	1	1	1	1	1	1	1	1	1	1	1	
1	0	0	1	0	1	1	1	1	1	1	0	1	1	1	
1	0	1	0	0	1	1	1	1	1	1	1	0	1	1	
1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	
1	1	0	0	0	1	1	1	1	1	1	1	1	0	1	
1	1	0	1	0	1	1	1	1	1	1	1	1	1	0	
1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	
1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	

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Stone's Theorem

$\mod 3 \ f_1$ in canonical POS

n_3	n_2	n_1	n_0	f_1	M_0	M_1	M_3	M_4	M_6	M_7	M_9	M_{10}	M_{12}	M_{13}	
0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	
0	0	0	1	0	1	0	1	1	1	1	1	1	1	1	
0	0	1	0	1	1	1	1	1	1	1	1	1	1	1	
0	0	1	1	0	1	1	0	1	1	1	1	1	1	1	
0	1	0	0	0	1	1	1	0	1	1	1	1	1	1	
0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	
0	1	1	0	0	1	1	1	1	0	1	1	1	1	1	
0	1	1	1	0	1	1	1	1	1	0	1	1	1	1	
1	0	0	0	1	1	1	1	1	1	1	1	1	1	1	
1	0	0	1	0	1	1	1	1	1	1	0	1	1	1	
1	0	1	0	0	1	1	1	1	1	1	1	0	1	1	
1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	
1	1	0	0	0	1	1	1	1	1	1	1	1	0	1	
1	1	0	1	0	1	1	1	1	1	1	1	1	1	0	
1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	
1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	

Boolean Algebra

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 $n ext{-Bits Binary}$ Adder

ield of Sets

tono's Theorem

$mod 3 f_1 in canonical POS$

n_3	n_2	n_1	n_0	f_1	M_0	M_1	M_3	M_4	M_6	M_7	M_9	M_{10}	M_{12}	M_{13}	
0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	
0	0	0	1	0	1	0	1	1	1	1	1	1	1	1	
0	0	1	0	1	1	1	1	1	1	1	1	1	1	1	
0	0	1	1	0	1	1	0	1	1	1	1	1	1	1	
0	1	0	0	0	1	1	1	0	1	1	1	1	1	1	
0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	
0	1	1	0	0	1	1	1	1	0	1	1	1	1	1	
0	1	1	1	0	1	1	1	1	1	0	1	1	1	1	
1	0	0	0	1	1	1	1	1	1	1	1	1	1	1	
1	0	0	1	0	1	1	1	1	1	1	0	1	1	1	
1	0	1	0	0	1	1	1	1	1	1	1	0	1	1	
1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	
1	1	0	0	0	1	1	1	1	1	1	1	1	0	1	
1	1	0	1	0	1	1	1	1	1	1	1	1	1	0	
1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	
1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	

Boolean Algebra

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Field of Sets

$mod 3 f_1 in canonical POS$

n_3	n_2	n_1	n_0	f_1	M_0	M_1	M_3	M_4	M_6	M_7	M_9	M_{10}	M_{12}	M_{13}	M_{15}
0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1
0	0	0	1	0	1	0	1	1	1	1	1	1	1	1	1
0	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1
0	0	1	1	0	1	1	0	1	1	1	1	1	1	1	1
0	1	0	0	0	1	1	1	0	1	1	1	1	1	1	1
0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1
0	1	1	0	0	1	1	1	1	0	1	1	1	1	1	1
0	1	1	1	0	1	1	1	1	1	0	1	1	1	1	1
1	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1
1	0	0	1	0	1	1	1	1	1	1	0	1	1	1	1
1	0	1	0	0	1	1	1	1	1	1	1	0	1	1	1
1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	0	0	0	1	1	1	1	1	1	1	1	0	1	1
1	1	0	1	0	1	1	1	1	1	1	1	1	1	0	1
1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	0

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Field of Sets

Stone's Theorem

$mod 3 f_1 in canonical POS$

n_3	n_2	n_1	n_0	f_1	M_0	M_1	M_3	M_4	M_6	M_7	M_9	M_{10}	M_{12}	M_{13}	M_{15}
0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1
0	0	0	1	0	1	0	1	1	1	1	1	1	1	1	1
0	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1
0	0	1	1	0	1	1	0	1	1	1	1	1	1	1	1
0	1	0	0	0	1	1	1	0	1	1	1	1	1	1	1
0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1
0	1	1	0	0	1	1	1	1	0	1	1	1	1	1	1
0	1	1	1	0	1	1	1	1	1	0	1	1	1	1	1
1	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1
1	0	0	1	0	1	1	1	1	1	1	0	1	1	1	1
1	0	1	0	0	1	1	1	1	1	1	1	0	1	1	1
1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	0	0	0	1	1	1	1	1	1	1	1	0	1	1
1	1	0	1	0	1	1	1	1	1	1	1	1	1	0	1
1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	0

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Field of Sets

Stone's Theorem

$\mod 3 \ f_1$ in canonical POS

n_3	n_2	n_1	n_0	f_1	M_0	M_1	M_3	M_4	M_6	M_7	M_9	M_{10}	M_{12}	M_{13}	M_{15}	$f_1(n_3, n_2, n_1, n_0) =$
0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	\prod (0, 1, 3, 4, 6, 7,
0	0	0	1	0	1	0	1	1	1	1	1	1	1	1	1	**
0	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	9, 10, 12, 13, 15) =
0	0	1	1	0	1	1	0	1	1	1	1	1	1	1	1	$(n_3+n_2+n_1+n_0)$.
0	1	0	0	0	1	1	1	0	1	1	1	1	1	1	1	$(n_3 + n_2 + n_1 + \bar{n}_0)$
0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	$(n_3 + n_2 + \bar{n}_1 + \bar{n}_0)$
0	1	1	0	0	1	1	1	1	0	1	1	1	1	1	1	
0	1	1	1	0	1	1	1	1	1	0	1	1	1	1	1	$(n_3 + \bar{n}_2 + n_1 + n_0)$.
1	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	$(n_3 + \bar{n}_2 + n_1 + n_0)$.
1	0	0	1	0	1	1	1	1	1	1	0	1	1	1	1	$(n_3 + \bar{n}_2 + \bar{n}_1 + \bar{n}_0)$.
1	0	1	0	0	1	1	1	1	1	1	1	0	1	1	1	$(\bar{n}_3 + n_2 + n_1 + \bar{n}_0)$
1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	$(\bar{n}_3 + n_2 + \bar{n}_1 + n_0)$
1	1	0	0	0	1	1	1	1	1	1	1	1	0	1	1	
1	1	0	1	0	1	1	1	1	1	1	1	1	1	0	1	$(\bar{n}_3 + \bar{n}_2 + n_1 + n_0)$.
1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	$(\bar{n}_3 + \bar{n}_2 + n_1 + \bar{n}_0)$.
1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	0	$(\bar{n}_3 + \bar{n}_2 + \bar{n}_1 + \bar{n}_0)$.

Unwinding

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$n \mod 3$ formulae

Boolean Algebras

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$$f_1 = \bar{n}_3 \bar{n}_2 n_1 \bar{n}_0 + \bar{n}_3 n_2 \bar{n}_1 n_0 + n_3 \bar{n}_2 \bar{n}_1 \bar{n}_0 + n_3 \bar{n}_2 n_1 n_0 + n_3 n_2 n_1 \bar{n}_0$$

$$f_0 = \bar{n}_3 \bar{n}_2 \bar{n}_1 n_0 + \bar{n}_3 n_2 \bar{n}_1 \bar{n}_0 + \bar{n}_3 n_2 n_1 n_0 + n_3 \bar{n}_2 n_1 \bar{n}_0 + n_3 n_2 \bar{n}_1 n_0$$

Note

In this case the POS form is uglier

Larger n's, kind of nasty

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mod 3 (notes)

• There is an algorithm for mod

- We used it to build the truth table
- We did **not** implement the algorithm
- Truth table implementation is mechanical
- However, it is not practical for large n's
- Usually it is also fast (relevant when we have hardware)
- Algorithm implementation is usually harder

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Sec. 12 The control

Truth tables do not scale up!!!!

However, they might give us more optimized functions

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(Carmi) Lecture 5 reached here

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Stone's Theorem

Addition

The plan:

- One bit adder (a.k.a Half Adder)
- Two bits adder
- Three bits adder?! Wrong direction
- Summing three bits (a.k.a Full Adder)
- *n*-Bits adder (a.k.a Binary Adder)

One Bit Adder

Boolean Algebras

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Problem

Devise a formula to add two one-bit numbers

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Devise a formula to add two one-bit numbers

- 2-bits inputs: Maximal possible sum is 2.
- Hence $f = \langle f_1, f_0 \rangle : \mathbb{B}^2 \to \mathbb{B}^2$

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Devise a formula to add two one-bit numbers

• 2-bits inputs: Maximal possible sum is 2.

• Hence $f = \langle f_1, f_0 \rangle : \mathbb{B}^2 \to \mathbb{B}^2$

		Decimal	Bin	ary
x	y	x+y	f_1	f_0
0	0	0	0	0
0	1	1	0	1
1	0	1	0	1
1	1	2	1	0

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Devise a formula to add two one-bit numbers

- 2-bits inputs: Maximal possible sum is 2.
- Hence $f = \langle f_1, f_0 \rangle : \mathbb{B}^2 \to \mathbb{B}^2$

		Decimal	Bin	ary
x	y	x+y	f_1	f_0
0	0	0	0	0
0	1	1	0	1
1	0	1	0	1
1	1	2	1	0

$$f_1 = xy$$

$$f_0 = x \oplus y$$

Addition

Half adder

Adder

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Problem

Devise a formula to calculate the sum of two numbers each in the range $0\!\!-\!\!3$

The function form

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Problem

Devise a formula to calculate the sum of two numbers each in the range $0\!\!-\!\!3$

The function form

• Of course we use binary coding

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Problem

Devise a formula to calculate the sum of two numbers each in the range $0\!\!-\!\!3$

The function form

- Of course we use binary coding
- Each of the inputs is 2-bits wide

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Problem

Devise a formula to calculate the sum of two numbers each in the range 0--3

The function form

- Of course we use binary coding
- Each of the inputs is 2-bits wide
- Thus sum is 6 at most

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Devise a formula to calculate the sum of two numbers each in the range 0--3

The function form

- Of course we use binary coding
- Each of the inputs is 2-bits wide
- Thus sum is 6 at most
- Thus the output is 3-bits wide

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Devise a formula to calculate the sum of two numbers each in the range 0--3

The function form

- Of course we use binary coding
- Each of the inputs is 2-bits wide
- Thus sum is 6 at most
- Thus the output is 3-bits wide
- The function is of the form $f = \langle f_2, f_1, f_0 \rangle : \mathbb{B}^4 \to \mathbb{B}^3$

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	Dec	imal			E	Binar			
x	y	x+y	x_1	x_0	y_1	y_0	f_2	f_1	f_0
0	0		0	0	0	0			
0	1		0	0	0	1			
0	2		0	0	1	0			
0	3		0	0	1	1			
1	0		0	1	0	0			
1	1		0	1	0	1			
1	2		0	1	1	0			
1	3		0	1	1	1			
2	0		1	0	0	0			
2 2 2 2 3	1		1	0	0	1			
2	2		1	0	1	0			
2	3		1	0	1	1			
3	0		1	1	0	0			
3	1		1	1	0	1			
3	2		1	1	1	0			
3	3		1	1	1	1			

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	Dec	imal			E	Binar	y		
x	y	x+y	x_1	x_0	y_1	y_0	f_2	f_1	f_0
0	0	0	0	0	0	0	0	0	0
0	1		0	0	0	1			
0	2		0	0	1	0			
0	3		0	0	1	1			
1	0		0	1	0	0			
1	1		0	1	0	1			
1	2		0	1	1	0			
1	3		0	1	1	1			
2	0		1	0	0	0			
2	1		1	0	0	1			
2	2		1	0	1	0			
2	3		1	0	1	1			
3	0		1	1	0	0			
3	1		1	1	0	1			
3	2		1	1	1	0			
3	3		1	1	1	1			

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	Dec	imal	Binary									
x	y	x+y	x_1	x_0	y_1	y_0	f_2	f_1	f_0			
0	0	0	0	0	0	0	0	0	0			
0	1	1	0	0	0	1	0	0	1			
0	2		0	0	1	0						
0	3		0	0	1	1						
1	0		0	1	0	0						
1	1		0	1	0	1						
1	2		0	1	1	0						
1	3		0	1	1	1						
2	0		1	0	0	0						
2	1		1	0	0	1						
2	2		1	0	1	0						
2	3		1	0	1	1						
3	0		1	1	0	0						
3	1		1	1	0	1						
3	2		1	1	1	0						
3	3		1	1	1	1						

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	Dec	imal			E	Binar	y		
x	y	x+y	x_1	x_0	y_1	y_0	f_2	f_1	f_0
0	0	0	0	0	0	0	0	0	0
0	1	1	0	0	0	1	0	0	1
0	2	2	0	0	1	0	0	1	0
0	3		0	0	1	1			
1	0		0	1	0	0			
1	1		0	1	0	1			
1	2		0	1	1	0			
1	3		0	1	1	1			
2	0		1	0	0	0			
2	1		1	0	0	1			
2	2		1	0	1	0			
2	3		1	0	1	1			
3	0		1	1	0	0			
3	1		1	1	0	1			
3	2		1	1	1	0			
3	3		1	1	1	1			

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	Dec	imal			E	Binar	y		
x	y	x+y	x_1	x_0	y_1	y_0	f_2	f_1	f_0
0	0	0	0	0	0	0	0	0	0
0	1	1	0	0	0	1	0	0	1
0	2	2	0	0	1	0	0	1	0
0	3	3	0	0	1	1	0	1	1
1	0		0	1	0	0			
1	1		0	1	0	1			
1	2		0	1	1	0			
1	3		0	1	1	1			
2	0		1	0	0	0			
2	1		1	0	0	1			
2	2		1	0	1	0			
2	3		1	0	1	1			
3	0		1	1	0	0			
3	1		1	1	0	1			
3	2		1	1	1	0			
3	3		1	1	1	1			

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	Dec	imal			E	3inar	y		
x	y	x+y	x_1	x_0	y_1	y_0	f_2	f_1	f_0
0	0	0	0	0	0	0	0	0	0
0	1	1	0	0	0	1	0	0	1
0	2	2	0	0	1	0	0	1	0
0	3	3	0	0	1	1	0	1	1
1	0	1	0	1	0	0	0	0	1
1	1		0	1	0	1			
1	2		0	1	1	0			
1	3		0	1	1	1			
2	0		1	0	0	0			
2	1		1	0	0	1			
2	2		1	0	1	0			
2	3		1	0	1	1			
3	0		1	1	0	0			
3	1		1	1	0	1			
3	2		1	1	1	0			
3	3		1	1	1	1			

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	Dec	imal			E	3inar	y		
x	y	x+y	x_1	x_0	y_1	y_0	f_2	f_1	f_0
0	0	0	0	0	0	0	0	0	0
0	1	1	0	0	0	1	0	0	1
0	2	2	0	0	1	0	0	1	0
0	3	3	0	0	1	1	0	1	1
1	0	1	0	1	0	0	0	0	1
1	1	2	0	1	0	1	0	1	0
1	2		0	1	1	0			
1	3		0	1	1	1			
2	0		1	0	0	0			
2	1		1	0	0	1			
2	2		1	0	1	0			
2	3		1	0	1	1			
3	0		1	1	0	0			
3	1		1	1	0	1			
3	2		1	1	1	0			
3	3		1	1	1	1			

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	Dec	imal			E	3inar	y		
x	y	x+y	x_1	x_0	y_1	y_0	f_2	f_1	f_0
0	0	0	0	0	0	0	0	0	0
0	1	1	0	0	0	1	0	0	1
0	2	2	0	0	1	0	0	1	0
0	3	3	0	0	1	1	0	1	1
1	0	1	0	1	0	0	0	0	1
1	1	2	0	1	0	1	0	1	0
1	2	3	0	1	1	0	0	1	1
1	3		0	1	1	1			
2	0		1	0	0	0			
2	1		1	0	0	1			
	2		1	0	1	0			
2 2	3		1	0	1	1			
3	0		1	1	0	0			
3	1		1	1	0	1			
3	2		1	1	1	0			
3	3		1	1	1	1			

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	Dec	imal			E	Binar	y		
x	y	x+y	x_1	x_0	y_1	y_0	f_2	f_1	f_0
0	0	0	0	0	0	0	0	0	0
0	1	1	0	0	0	1	0	0	1
0	2	2	0	0	1	0	0	1	0
0	3	3	0	0	1	1	0	1	1
1	0	1	0	1	0	0	0	0	1
1	1	2	0	1	0	1	0	1	0
1	2	3	0	1	1	0	0	1	1
1	3	4	0	1	1	1	1	0	0
2	0		1	0	0	0			
2	1		1	0	0	1			
2	2		1	0	1	0			
2	3		1	0	1	1			
3	0		1	1	0	0			
3	1		1	1	0	1			
3	2		1	1	1	0			
3	3		1	1	1	1			

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	Dec	imal			E	Binar	y		
x	y	x+y	x_1	x_0	y_1	y_0	f_2	f_1	f_0
0	0	0	0	0	0	0	0	0	0
0	1	1	0	0	0	1	0	0	1
0	2	2	0	0	1	0	0	1	0
0	3	3	0	0	1	1	0	1	1
1	0	1	0	1	0	0	0	0	1
1	1	2	0	1	0	1	0	1	0
1	2	3	0	1	1	0	0	1	1
1	3	4	0	1	1	1	1	0	0
2	0	2	1	0	0	0	0	1	0
2	1		1	0	0	1			
2	2		1	0	1	0			
2	3		1	0	1	1			
3	0		1	1	0	0			
3	1		1	1	0	1			
3	2		1	1	1	0			
3	3		1	1	1	1			

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	Dec	imal			Е	3inar	y		
x	y	x+y	x_1	x_0	y_1	y_0	f_2	f_1	f_0
0	0	0	0	0	0	0	0	0	0
0	1	1	0	0	0	1	0	0	1
0	2	2	0	0	1	0	0	1	0
0	3	3	0	0	1	1	0	1	1
1	0	1	0	1	0	0	0	0	1
1	1	2	0	1	0	1	0	1	0
1	2	3	0	1	1	0	0	1	1
1	3	4	0	1	1	1	1	0	0
2	0	2	1	0	0	0	0	1	0
2	1	3	1	0	0	1	0	1	1
2	2		1	0	1	0			
2	3		1	0	1	1			
3	0		1	1	0	0			
3	1		1	1	0	1			
3	2		1	1	1	0			
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	De	ec	imal			E	3inar	y		
a	: y		x + y	x_1	x_0	y_1	y_0	f_2	f_1	f_0
(0		0	0	0	0	0	0	0	0
() 1		1	0	0	0	1	0	0	1
() 2		2	0	0	1	0	0	1	0
() 3		3	0	0	1	1	0	1	1
1	. 0		1	0	1	0	0	0	0	1
1	. 1		2	0	1	0	1	0	1	0
1	. 2		3	0	1	1	0	0	1	1
1	. 3		4	0	1	1	1	1	0	0
2	2 0		2	1	0	0	0	0	1	0
2	2 1		3	1	0	0	1	0	1	1
2	2 2		4	1	0	1	0	1	0	0
2	2 3			1	0	1	1			
3	3 0			1	1	0	0			
3	3 1			1	1	0	1			
3	3 2			1	1	1	0			
3	3			1	1	1	1			

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	Dec	imal			E	Binar	y		
x	y	x+y	x_1	x_0	y_1	y_0	f_2	f_1	f_0
0	0	0	0	0	0	0	0	0	0
0	1	1	0	0	0	1	0	0	1
0	2	2	0	0	1	0	0	1	0
0	3	3	0	0	1	1	0	1	1
1	0	1	0	1	0	0	0	0	1
1	1	2	0	1	0	1	0	1	0
1	2	3	0	1	1	0	0	1	1
1	3	4	0	1	1	1	1	0	0
2	0	2	1	0	0	0	0	1	0
2	1	3	1	0	0	1	0	1	1
	2	4	1	0	1	0	1	0	0
2 2	3	5	1	0	1	1	1	0	1
3	0		1	1	0	0			
3	1		1	1	0	1			
3	2		1	1	1	0			
3	3		1	1	1	1			

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	Dec	imal			E	Binar	y		
x	y	x+y	x_1	x_0	y_1	y_0	f_2	f_1	f_0
0	0	0	0	0	0	0	0	0	0
0	1	1	0	0	0	1	0	0	1
0	2	2	0	0	1	0	0	1	0
0	3	3	0	0	1	1	0	1	1
1	0	1	0	1	0	0	0	0	1
1	1	2	0	1	0	1	0	1	0
1	2	3	0	1	1	0	0	1	1
1	3	4	0	1	1	1	1	0	0
2	0	2	1	0	0	0	0	1	0
2	1	3	1	0	0	1	0	1	1
2 2	2	4	1	0	1	0	1	0	0
	3	5	1	0	1	1	1	0	1
3	0	3	1	1	0	0	0	1	1
3	1		1	1	0	1			
3	2		1	1	1	0			
3	3		1	1	1	1			

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	Dec	imal			E	Binar	y		
x	y	x+y	x_1	x_0	y_1	y_0	f_2	f_1	f_0
0	0	0	0	0	0	0	0	0	0
0	1	1	0	0	0	1	0	0	1
0	2	2	0	0	1	0	0	1	0
0	3	3	0	0	1	1	0	1	1
1	0	1	0	1	0	0	0	0	1
1	1	2	0	1	0	1	0	1	0
1	2	3	0	1	1	0	0	1	1
1	3	4	0	1	1	1	1	0	0
2	0	2	1	0	0	0	0	1	0
2	1	3	1	0	0	1	0	1	1
2	2	4	1	0	1	0	1	0	0
2	3	5	1	0	1	1	1	0	1
3	0	3	1	1	0	0	0	1	1
3	1	4	1	1	0	1	1	0	0
3	2		1	1	1	0			
3	3		1	1	1	1			

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	Dec	imal			E	Binar	y		
x	y	x+y	x_1	x_0	y_1	y_0	f_2	f_1	f_0
0	0	0	0	0	0	0	0	0	0
0	1	1	0	0	0	1	0	0	1
0	2	2	0	0	1	0	0	1	0
0	3	3	0	0	1	1	0	1	1
1	0	1	0	1	0	0	0	0	1
1	1	2	0	1	0	1	0	1	0
1	2	3	0	1	1	0	0	1	1
1	3	4	0	1	1	1	1	0	0
2	0	2	1	0	0	0	0	1	0
2	1	3	1	0	0	1	0	1	1
2	2	4	1	0	1	0	1	0	0
2	3	5	1	0	1	1	1	0	1
3	0	3	1	1	0	0	0	1	1
3	1	4	1	1	0	1	1	0	0
3	2	5	1	1	1	0	1	0	1
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	Dec	imal			Е	3inar	y		
x	y	x+y	x_1	x_0	y_1	y_0	f_2	f_1	f_0
0	0	0	0	0	0	0	0	0	0
0	1	1	0	0	0	1	0	0	1
0	2	2	0	0	1	0	0	1	0
0	3	3	0	0	1	1	0	1	1
1	0	1	0	1	0	0	0	0	1
1	1	2	0	1	0	1	0	1	0
1	2	3	0	1	1	0	0	1	1
1	3	4	0	1	1	1	1	0	0
2	0	2	1	0	0	0	0	1	0
2	1	3	1	0	0	1	0	1	1
2	2	4	1	0	1	0	1	0	0
2	3	5	1	0	1	1	1	0	1
3	0	3	1	1	0	0	0	1	1
3	1	4	1	1	0	1	1	0	0
3	2	5	1	1	1	0	1	0	1
3	3	6	1	1	1	1	1	1	0

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	Dec	imal			E	Binar	y		
x	y	x+y	x_1	x_0	y_1	y_0	f_2	f_1	f_0
0	0	0	0	0	0	0	0	0	0
0	1	1	0	0	0	1	0	0	1
0	2	2	0	0	1	0	0	1	0
0	3	3	0	0	1	1	0	1	1
1	0	1	0	1	0	0	0	0	1
1	1	2	0	1	0	1	0	1	0
1	2	3	0	1	1	0	0	1	1
1	3	4	0	1	1	1	1	0	0
2	0	2	1	0	0	0	0	1	0
2	1	3	1	0	0	1	0	1	1
2	2	4	1	0	1	0	1	0	0
2	3	5	1	0	1	1	1	0	1
3	0	3	1	1	0	0	0	1	1
3	1	4	1	1	0	1	1	0	0
3	2	5	1	1	1	0	1	0	1
3	3	6	1	1	1	1	1	1	0

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$f_{2} = \bar{x}_{1}x_{0}y_{1}y_{0} + x_{1}\bar{x}_{0}y_{1}\bar{y}_{0} + x_{1}\bar{x}_{0}y_{1}y_{0} + x_{1}x_{0}\bar{y}_{1}y_{0} + x_{1}x_{0}\bar{y}_{1}y_{0} + x_{1}x_{0}y_{1}\bar{y}_{0} + x_{1}x_{0}y_{1}y_{0} + \bar{x}_{1}x_{0}y_{1}\bar{y}_{0} + \bar{x}_{1}\bar{x}_{0}y_{1}\bar{y}_{0} + \bar{x}_{1}x_{0}\bar{y}_{1}y_{0} + \bar{x}_{1}x_{0}y_{1}\bar{y}_{0} + x_{1}\bar{x}_{0}\bar{y}_{1}\bar{y}_{0} + x_{1}x_{0}\bar{y}_{1}\bar{y}_{0} + x_{1}x_{0}y_{1}y_{0} + \bar{x}_{1}x_{0}y_{1}\bar{y}_{0} + \bar{x}_{1}\bar{x}_{0}y_{1}\bar{y}_{0} + \bar{x}_{1}x_{0}y_{1}\bar{y}_{0} + \bar{x}_{1}x_{0}\bar{y}_{1}\bar{y}_{0} + \bar{x}_{1}x_{0}\bar{y}_{1}\bar{y}_{0} + \bar{x}_{1}x_{0}\bar{y}_{1}\bar{y}_{0} + \bar{x}_{1}x_{0}\bar{y}_{1}\bar{y}_{0} + \bar{x}_{1}x_{0}\bar{y}_{1}\bar{y}_{0} + \bar{x}_{1}x_{0}\bar{y}_{1}\bar{y}_{0} + \bar{x}_{$

 $x_1\bar{x}_0\bar{y}_1y_0 + x_1\bar{x}_0y_1y_0 + x_1x_0\bar{y}_1\bar{y}_0 + x_1x_0y_1\bar{y}_0$

The notes of mod 3 hold also here

More than Two Bits Adder

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Stone's Theorem

• Three bits: $f: \mathbb{B}^6 \to \mathbb{B}^4$

- The general case looks hopeless
- However, we have seen how to add numbers:
 - ► Long addition of representations
- Until now our method was as follows:
 - ▶ Use an algoritm to generate truth table
 - ► Generate a formula from the truth table

The exponential explosion requires something else

• The formula we generate will implement the algorithm

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			0
x_3	x_2	x_1	x_0
 y_3	y_2	y_1	y_0

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			0	I	0
			Ü		U
x_3	x_2 y_2	x_1	x_0		x_0
$\underline{}$ y_3	y_2	y_1	y_0		y_0
				c_0	z_0

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		c_0			0
x_3	x_2 y_2	x_1	x_0		x_0
y_3	y_2	y_1	y_0		y_0
			z_0	c_0	z_0

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				ı	
		c_0			c_0
x_3	x_2	x_1 y_1	x_0		x_1
 y_3	y_2	y_1	y_0		y_1
			z_0	c_1	z_1

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	c_1				c_0
x_3	x_2	x_1 y_1	x_0		x_1
y_3	y_2		y_0		y_1
		z_1	z_0	c_1	z_1

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	c_1				c_1
x_3	x_2 y_2	x_1	x_0		x_2
y_3	y_2	y_1	y_0		y_2
		z_1	z_0	c_2	z_2

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c_2					c_1
x_3	x_2 y_2	x_1	x_0		x_2
$\underline{}$	y_2	y_1	y_0		y_2
	z_2	z_1	z_0	c_2	z_2

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c_2					c_2
x_3	x_2 y_2	x_1	x_0		x_3
 y_3					y_3
	z_2	z_1	z_0	c_3	z_3

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Field of Sets

c_3	c_2					c_2
	x_3	x_2 y_2	x_1	x_0		x_3
	y_3	y_2	y_1	y_0		y_3
	z_3	z_2	z_1	z_0	c_3	z_3

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c_3	c_2			
	x_3	x_2	x_1	x_0
	y_3	y_2	y_1	y_0
	z_3	z_2	z_1	z_0

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- Unsigned overflow: $c_3 = 1$
- Signed overflow: $c_3 \neq c_2$

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- Unsigned overflow: $c_3 = 1$
- Signed overflow: $c_3 \neq c_2$

Corollary

There is a function f such that for each n,

$$\langle c_n, z_n \rangle = f(c_{n-1}, x_n, y_n),$$

where
$$c_{-1} = 0$$

Summing Three Bits (full adder)

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Problem

Devise a formula to sum three bits

The form of the function

- Three bits input
- The sum is at most three
- Thus the output is two bits
- $f = \langle f_1, f_0 \rangle : \mathbb{B}^3 \to \mathbb{B}^2$

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			Decimal	Binary	
\boldsymbol{x}	y	z	x+y+z	f_1	f_0
0	0	0			
0	0	1			
0	1	0			
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			

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			Decimal	Bin	ary
\boldsymbol{x}	y	z	x+y+z	f_1	f_0
0	0	0	0	0	0
0	0	1			
0	1	0			
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			

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			Decimal	Bin	ary
\boldsymbol{x}	y	z	x+y+z	f_1	f_0
0	0	0	0	0	0
0	0	1	1	0	1
0	1	0			
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			

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			Decimal	Bin	ary
\boldsymbol{x}	y	z	x+y+z	f_1	f_0
0	0	0	0	0	0
0	0	1	1	0	1
0	1	0	1	0	1
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			

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			Decimal	Bin	ary
\boldsymbol{x}	y	z	x+y+z	f_1	f_0
0	0	0	0	0	0
0	0	1	1	0	1
0	1	0	1	0	1
0	1	1	2	1	0
1	0	0			
1	0	1			
1	1	0			
1	1	1			

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			Decimal	Binary	
\boldsymbol{x}	y	z	x+y+z	f_1	f_0
0	0	0	0	0	0
0	0	1	1	0	1
0	1	0	1	0	1
0	1	1	2	1	0
1	0	0	1	0	1
1	0	1			
1	1	0			
1	1	1			

Boolean Algebras

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0	0	1	1	0	1
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0	1	1	2	1	0
1	0	0	1	0	1
1	0	1	2	1	0
	0 0 0	0 0 0 0 0 1	0 0 0 0 0 1 0 1 0	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$

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Stone's Theorem

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Addition of Three Bits (formula)

				Decimal	Binary	
	\boldsymbol{x}	y	z	x+y+z	f_1	f_0
	0	0	0	0	0	0
	0	0	1	1	0	1
	0	1	0	1	0	1
	0	1	1	2	1	0
ĺ	1	0	0	1	0	1
	1	0	1	2	1	0
ĺ	1	1	0	2	1	0
	1	1	1	3	1	1

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1	1	0	2	1	0
1	1	1	3	1	1

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			Decimal	Binary	
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0	0	1	1	0	1
0	1	0	1	0	1
0	1	1	2	1	0
1	0	0	1	0	1
1	0	1	2	1	0
1	1	0	2	1	0
1	1	1	3	1	1

- $f_0 = x \oplus y \oplus z$
- $f_1(x, y, z) = \sum (3, 5, 6, 7) = \prod (0, 1, 2, 4)$

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$$f_1 = \bar{x}yz + x\bar{y}z + xy\bar{z} + xyz =$$

$$= (\bar{x} + x)yz + (\bar{y} + y)xz + xy(\bar{z} + z) =$$

$$= yz + xz + xy$$

Binary Adder without Truth Tables

Well, almost, we use the full adder

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Problem

For each n devise a function to compute the sum of two numbers each in the range $0-2^n-1$

Function Form

- Of course, binary
- Each of the input numbers is *n*-bits wide
- The output is n+1-bits wide
- The form is $f = \langle f_n, \cdots, f_0 \rangle : \mathbb{B}^{2n} \to \mathbb{B}^{n+1}$

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Definitions

- Let $x_{n-1} \cdots x_0$ and $y_{n-1} \cdots y_0$ be the binary representation of the two input numbers
- Let $z_n \cdots z_0$ be the binary representation of the sum
- Let $f = \langle f_1, f_0 \rangle : \mathbb{B}^3 \to \mathbb{B}^2$ be the full adder

 $\langle c_0, z_0 \rangle = \langle f_1(x_0, y_0, 0), f_0(x_0, y_0, 0) \rangle$

Long addition

$$\langle c_1, z_1 \rangle = \langle f_1(x_1, y_1, c_0), f_0(x_1, y_1, c_0) \rangle$$

$$\vdots = \vdots$$

$$\langle c_{n-1}, z_{n-1} \rangle = \langle f_1(x_{n-1}, y_{n-1}, c_{n-2}), f_0(x_{n-1}, y_{n-1}, c_{n-2}) \rangle$$

$$z_n = c_{n-1}$$

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Define the functions q_0 and q_1

$$g_0(k, x_k, y_k) = \begin{cases} f_0(x_k, y_k, 0) & k = 0, \\ f_0(x_k, y_k, g_1(k - 1, x_{k-1}, y_{k-1})) & 0 < k < n, \\ f_0(0, 0, g_1(n - 1, x_{n-1}, y_{n-1})) & k = n \end{cases}$$

$$g_1(k, x_k, y_k) = \begin{cases} f_1(x_k, y_k, 0) & k = 0, \\ f_1(x_k, y_k, g_1(k - 1, x_{k-1}, y_{k-1})) & 0 < k < n \end{cases}$$

The following holds

For each $i \leq n$, $z_i = g_0(i, x_i, y_i)$

q_1 is going to take long to compute

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Extras

In the following we deal with arbitrary boolean algebras

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Field of Sets

Definition

The structure $\langle P,\emptyset,X,\cup,\cap,\setminus\rangle$, where $P\subseteq\mathcal{P}(X)$, is a field of sets if the following hold for each $x,y\in P\colon\emptyset\in P,\,x\cup y\in P,\,x\cap y\in P$, and $X\setminus x\in P$. (By \ we mean the operation $X\setminus x$.)

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Lemma

The structure $\langle P, \emptyset, X, \cup, \cap, \setminus \rangle$, where $P \subseteq \mathcal{P}(X)$ is a field of sets, is a boolean algebra.

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Proof.

There is really nothing to prove here assuming one understands the meaning of the set operations \cup , \cap and \setminus .

- 1. $x \cup \emptyset = x$ and $x \cap X = x$.
- 2. $x \cup y = y \cup x$ and $x \cap y = y \cap x$.
- 3. $(x \cup y) \cup z = x \cup (y \cup z)$ and $(x \cap y) \cap z = x \cap (y \cap z)$.
- 4. $x \cup (y \cap z) = (x \cup y) \cap (x \cup z)$ and $x \cap (y \cup z) = (x \cap y) \cup (x \cap z)$.
- 5. $x \cup (X \setminus x) = X$ and $x \cap (A \setminus x) = \emptyset$.

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Corollary

For each set X, the structure $\langle \mathcal{P}(X), \emptyset, X, \cup, \cap, \setminus \rangle$ is a boolean algebra.

Taking X to be the empty set in the above corollary we have $\mathcal{P}(X) = \{\emptyset, \{\emptyset\}\}$! Thus we got the 2-valued boolean algebra!! This might lead us to suspect (correctly!) that somehow every boolean algebra can be realized as a field of sets with the usual set operations!

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Field of Sets

Theorem (Stone representation theorem [?])

Each boolean algebra is isomorphic to a field of sets.

Definition

Let \mathbb{B} be a boolean algebra.

- 1. We will say that $x \leq y$ if x = xy.
- 2. A subset $U \subsetneq \mathbb{B}$ is called an ultrafilter if the following hold:
 - 2.1 $0 \notin U$ and $1 \in U$.
 - 2.2 If $x \in U$ and $x \leq y$ then $y \in U$.
 - 2.3 If $x, y \in U$ then $x \cdot y \in U$.
 - 2.4 If $x \in B$ then either $x \in U$ or $\bar{x} \in U$.

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Proof.

Let $\mathbb B$ be a boolean algebra. Let $X=\{U\mid U \text{ is an ultrafilter over }\mathbb B\}$. For each $b\in\mathbb B$ let $\mathcal U_b=\{U\in X\mid b\in U\}$. Let $P=\{\mathcal U_b\mid b\in\mathbb B\}$. Then $P\subseteq\mathcal P(X)$ and $\langle P,\emptyset,X,\cup,\cap,\setminus\rangle$ is a field of sets.

Define the function $\pi:\mathbb{B}\to P$ by letting $\pi(b)=\mathcal{U}_b$ for each $b\in\mathbb{B}$. It is not hard to check that

$$\pi: \langle \mathbb{B}, 0, 1, +, \cdot, \bar{} \rangle \to \langle P, \emptyset, X, \cup, \cap, \backslash \rangle$$

is an isomorphism.

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